RELATIVITY AND THE UNIVERSE

A POPULAR INTRODUCTION INTO EINSTEIN'S THEORY OF SPACE & TIME

BY

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AUTHORISED TRANSLATION BY

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WITH FIVE DIAGRAMS

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TO

ALBERT EINSTEIN

IN TOKEN

OF THE AUTHOR'S

REVERENT ADMIRATION

PREFACE TO THE ENGLISH EDITION

WHILE the first English edition of my little book, *Das Weltbild der Relativitätstheorie*, was in course of publication, it was found necessary to prepare a third German edition. However, since the text of the second edition, apart from the correction of a few minor errors, will remain practically unchanged there is no reason why the English translation should not be based upon it.

I have to express my thanks to Dr. Karl Wichmann for kindly offering me an opportunity of inserting the corrections mentioned. It would cause me much pleasure if this English edition should contribute to the great object of spreading, as widely as possible, a knowledge of the fundamental ideas of Einstein's theory of relativity.

HARRY SCHMIDT

ALTONA (ELBE)
April 14, 1921

TRANSLATOR'S NOTE

CONSIDERING the widespread interest aroused in Einstein's theory of relativity in the English-speaking world, the translation of this little book hardly seems to need a justification. The book is full of suggestive thought, and in simple, non-technical language the author attempts to explain how Einstein arrived at his conclusions, and how, if we accept them, they are likely to modify our view of the universe. Since the author presumes no acquaintance with mathematics and theoretical physics the ordinary reader, after a study of the book, should feel himself in a position to form an independent opinion about the problems raised by this new and startling theory of space and time.

I desire to offer my sincerest thanks to Dr. Robert W. Lawson, of the University of Sheffield, for kindly reading through the manuscript, and for suggesting numerous and valuable improvements, as well as to Professor H. G. Fiedler, M.A., Ph.D., of the University of Oxford, for his helpful advice with regard to the translation of the poetical passages occurring in the book.

KARL WICHMANN

SHEFFIELD
May 17, 1921

INTRODUCTORY NOTE

THIS little book is the outcome of a series of tutorial lectures delivered in connection with the Free Extension Courses organised by the town of Altona. The vivid interest shown in the subject by an audience, drawn from all sorts of professions and callings, induced me to write them out. In order to preserve the original character of the lectures I decided to treat the matter in a colloquial manner. I have thus had an opportunity of putting before the reader's mind, step by step, and in leisurely fashion, the very considerable difficulties which, as we know, are connected with the theory of relativity.

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RELATIVITY AND THE UNIVERSE

I

THE VIEW OF THE UNIVERSE ACCORDING TO MODERN PHYSICS

A RADIANT day of May, as perfect as you could imagine it, is drawing to a close. Behind the houses of Störort, which descend almost to the edge of the water on the promontory between the rivers Elbe and Stör, the sky is glowing in flaming red like a sea of fire. A glittering streak, tinged with carmine, narrow and sharp like a mathematical line—the Elbe—forms its lower boundary. In an upward direction the sky merges into a chaos of all sorts of clouds. Cirro-cumulus and cumulus clouds, showing a thousand different forms, are massed together. From deep red the wild play of colours passes to pale pink and sulphur-like yellow, to moss-green and deep dark blue. The three buildings in front of us, picturesquely nestling in groups of leafy trees, appear to be veiled in purple mist, and the carpet of the meadows, interwoven with flowers, is glimmering in a mysterious twilight. Above us, in lofty heights, and behind us where evening stretches his feelers out of the western horizon our eye is enjoying the delicate counter-glow of the symphony of colours

dying away in the east. And round about us the waters of the Stör are undulating in shimmering rhythm, hardly noticeable as they play round our little boat, which leisurely glides upstream and follows the gentle impulse of the tide.

A sacred hour in Nature, this setting of the sun. Full of longing our eyes follow the disappearing orb of day, which offers us so much beauty in its farewell greeting. In silent awe you look at the glorious scene. Eye and heart are revelling in blissful intoxication. The idea of an all-powerful harmony of the universe unconsciously enters our mind.

What a strange thing it seems, this harmony of the universe. The harmony of the infinite in great things and small.

The earth on which we are living—a sphere having a surface of about 150,000,000 square kilometres—and to which we are confined, has been considerably reduced as regards its importance for the whole, although it is considered by the man in the street as the " world " pure and simple. It has been found to be a satellite of the sun, just like those planets which for many, many years have attracted the special attention of the astronomers. It was Nikolaus Kopernikus who taught us the structure of the solar system, who appointed to the earth and the planets their firmly established circular orbits round the solar centre. At the beginning of the seventeenth century Johannes Kepler derived the accurate laws of planetary motion from the mass of observations which Tycho Brahe had accumulated with bee-like industry. The satellites of the sun revolve round the burning central star in ellipses, and not in circles. The sun is not fixed at their centre, but a little away from it (at one of the so-called foci of each of these ellipses). The planets do not travel in their orbits with permanently uniform velocity, but when near to the sun they move faster than when they are farther away.

And the rate of change of these velocities is regulated in a peculiar manner. For if you imagine any of the planets, say the earth, joined to the sun by a straight line, this line would obviously describe a sort of triangular surface owing to the planet's motion. The altitude of this surface corresponds at each moment to the distance between the earth and the sun; the base equals the length of path the earth has travelled during the particular time. The area of the surface described naturally depends on its altitude and base. Now the motion of each individual planet is regulated in such a manner that wherever it may be in its orbit the straight line joining it to the sun will pass over equal sections of the surface during equal periods of time. In these circumstances you will easily understand that the planet, when near the sun, *i.e.* when the altitude is less, will have to accelerate its speed in a very definite manner, and thus will have to travel a greater distance within a given time than when it is far away from the sun, so that equally large surfaces may be covered in either case. These rules about planetary motion are completed by a further law which expresses certain relations existing between the period of revolution of the various planets and their average distances from the solar centre; but we need not trouble about it here.

Hearing these laws mentioned for the first time, you will undoubtedly feel inclined to believe that they are based on several causes different from each other. And here the lasting achievement of Newton comes in, the English physicist whose genius discovered by mere calculation a marvellous connection between them. For, as a result of mathematical calculations, the details of which I am only too glad to spare you, he was able to demonstrate that all the laws I have just mentioned can be derived as inevitable consequences of one fundamental law. This fundamental law is the so-called law of gravitation, according to which all bodies exert

a mutual attraction on each other, the intensity of which depends simply and solely on the mass of the bodies, and their distances from each other. The greater the masses the greater becomes the force of attraction ; but the farther they are distant from each other the less they act upon each other. Since this simple law applies to all bodies it becomes t h e law of the universe. It compels the planets to travel round the sun in ellipses, just as it compels the moons which have become known to us as faithful satellites of various planets. It regulates the courses of the comets—these tramps of the universe ; and when the astronomer at his telescope investigates the motions of the "fixed" stars he uses the law of gravitation as his safe guide. Gravitation draws the falling stone towards the earth, and when C a v e n d i s h, in 1798, attached a piece of metal shaped somewhat like dumb-bells to a thin wire between two heavy balls of lead, the way in which the dumb-bells turned enabled him to ascertain, by direct observation, the action of the force of attraction. Thus the discovery of the law of gravitation is a scientific achievement of the first rank, and the name of Newton, who formulated it, will be indelibly inscribed in the annals of the history of science.

But now look at the disc of the sun over there, appearing gigantic in size, how it prepares to plunge rapidly below the horizon. Its last rays, as if bidding us farewell, are flitting through the landscape all round. All sorts of scenes from ancient Greek mythology are rising in your memory. Helios, the god of the sun, the son of Hyperion and Theia, turns his golden chariot towards the Okeanos, having completed his day's journey. Eos, the "rosy-fingered," the charming goddess of the red morning and evening, drives along in her floating red-yellow garments, swinging a flaming torch in her right hand. No doubt we men of modern times no longer feel quite at ease with these poetical interpretations. Our science forbids us to take them

seriously. But we, too, are powerfully affected by the charm of this wonderful view, and, standing in raptures, we think of Faust's words:

> Slow sinks the orb, the day is now no more;
> Yonder he hastens to diffuse new life.
> (*Translated by Anna Swanwick*)

As a member of the solar system our earth is placed in a space extending without boundaries. When, a few hours from now, the twilight will have given way to the night, the innumerable host of stars will adorn the dark tent of heaven. The light of these stars has carried to us strange news out of the far distance. It has told us of thousands and thousands of glimmering suns round which their planets are revolving similar to the planets of our sun. The earth is separated from her sun by 150,000,000 kilometres; in order to cover that distance, you would have to place the length of her equator, end to end, about 3750 times. But what is this in comparison with the distances of the fixed stars, whose light takes decades, or centuries, if not thousands of years, to reach our earth? Light, in a single second, travels a distance of 300,000 kilometres; more than four years have to elapse before it can reach us from the star nearest to us! More than four years, each of them containing far more than 31,000,000 seconds! Thus our earth seems a grain of sand in the universe, and an infinity is revealed to us that fills us with awe. And yet this is not the only infinity which the human brain has disclosed in its restless subtle speculations.

As you know from your chemistry lessons, the multiplicity of the matter which is all round us rests finally on a few fundamental substances called chemical elements. You have heard of the historical development of this conception. Of old Thales of Miletos and his element—water. Of Empedocles with his four elements, which the learned Aristotle knew

how to endow with such authority, and which our poet Schiller has branded as fierce enemies of the work created by lofty human endeavour. Perhaps you have heard, too, of the philosophy of the alchemists in the Middle Ages who tried to raise sulphur and mercury to the dignity of original constituents of all metals. And then you have heard of the awakening of chemistry as an exact science, followed soon by an insight into the constitution of matter. All material substances were conceived as various combinations of chemical elements which, in themselves, are indivisible and unchanging. In his classical textbook of chemistry Antoine Laurent Lavoisier has expressed this state of things in the following words: " People will perhaps be surprised to find in this elementary book on chemistry no chapter dealing with the primitive constituents or elements of bodies; but, with respect to this point, I have to remark that this tendency to demand that all bodies in nature should be composed of three or four elements, only dates from a prejudice which we originally owe to the Greek philosophers. The assumption of four elements, which by their varying relations constitute all bodies known to us, is a mere hypothesis. . . . But if we connect with the term element or fundamental substance of bodies, the conception of the highest aim which chemical analysis has reached, then all substances which we have not been able to analyse in any way are elements for us. . . . They act before our eyes as simple bodies, and we have no right to consider them as compounds as long as we have got no evidence to this effect by experience and observation."

Research during the nineteenth century has strictly adhered to this definition of chemical elements. It was supplemented in essential points by the atomic theory established by Dalton. According to this theory a given quantity of an element cannot be divided

into as many parts as you like. Just as I can separate from a bag full of peas numerous quantities, none of which can be smaller than a single pea—so in the same way, in this case, too, a continued division leads to smallest particles, which are called atoms. The atoms of a given element are absolutely alike, both in their properties, and above all, in their weight; on the other hand, the atoms of different elements are distinguished from each other in exactly the same way as larger, visible quantities of them, and, moreover, they vary in weight. However, it is not so easy to determine the true weight of individual atoms; but chemistry has discovered means by which the relative weight of the atoms with regard to each other may be determined.

No doubt this illustration with the peas which I gave just now has shocked you a little. I do not mean to say that this comparison is bound to be deficient owing to the considerable difference in size shown by the individual peas; we might easily get over this difficulty by assuming from the beginning that all the peas were exactly of the same size. Much more important seems to be the following consideration: When picking out the peas from the bag I certainly cannot arrive at smaller quantities than a single pea. But these individual peas can—*e.g.* by using a knife—be subdivided into much smaller parts without any difficulty. Do similar considerations not apply to atoms? This would be quite conceivable even without necessitating any alteration of the interpretation we gave to the conception of an atom. For the fragments produced by such an operation would no longer possess all the qualities of the particular chemical element, just as a fragment of a pea no longer possesses the full character of a pea, or as a chair that has lost a leg is no longer a real chair. In other words, in this case atoms would have to be conceived as the ultimate and smallest bearers of the chemical properties exhibited by different elements.

As a matter of fact, modern research has led to the very interesting result that atoms are built up out of simpler constituents. These constituents, as far as their nature is concerned, are the same for all the atoms hitherto known. They are, on the one hand, negative electric charges, the so-called electrons, and, on the other hand, the smallest particles of matter carrying a positive electric charge. What seems most remarkable in all this, however, is the fact that each individual atom represents a structure somewhat like a solar system. Its centre is formed by a nucleus of matter charged with positive electricity round which the negative electrons are constantly travelling in closed orbits. An individual electron is about a thousand billion times smaller than the smallest known atom; and the size of the atomic nucleus is again about one-thousand millionth part of the size of a negative electron. Thus the vast picture which astronomy has sketched of the realm of heavenly bodies recurs in the world of atoms on an infinitely reduced scale. It even appears as if this similarity is something more than merely external! For Sommerfeld, applying Kepler's above-mentioned laws of planetary motion to the electrons revolving in an atom of hydrogen, was able, by purely mathematical considerations, to arrive at results which are in complete harmony with facts of experience well established by physical experiments.

But this by no means exhausts the marvels of the world of atoms. For the most recent considerations have led us to the view that the minute atomic nucleus —this fraction of an electron!—possesses a highly complicated structure. It possibly represents a negative electric ball in the interior of which particles are moving charged with positive electricity. And the quantity of the positive charges enclosed in it outweighs by a definite amount the negative charge of the ball, so that the whole may act upon its surroundings as a uniform positive

nucleus. In this case the atom of the chemist, a mere nothing in the universe of the stars, would be a solar system in a twofold sense. On the one hand, the positive nucleus would be the sun having the negative electrons as faithful satellites. On the other hand, the nucleus would be a world in itself where positive electrons revolve in their circles. Eternal unbending laws are governing both worlds—that of the stars as well as that of the electrons. And in these laws we see the two infinities of the great and the small meeting each other.

Nearly twenty-five centuries have elapsed since the Greek philosopher, Democritus, astonished his fellow-citizens at Abdera in Asia Minor by an extremely strange theory. In bold speech he declared the whole of nature which surrounds us to be an illusion of our senses. This diversity of colour and form, with its abundance of individual phenomena restlessly following one upon the other, became to him a deceptive world of appearance, without the faintest claim to real existence, a world shaped by the human brain alone, because eye and ear, nose and tongue, and the groping hands, are trying incessantly to clothe in misleading garments the impressions reaching us from the outer world. In reality nothing exists but atoms, smallest particles of varying size and shape, but in all other respects entirely alike. They alone build up the world in which we are living, they alone deceive our senses by the confused illusion of events in nature which the unsophisticated mind would like to accept at their face-value. These atoms rushing about in the maddest motions cause the eye to "see" light and colour, or the ear to "hear" sounds and noises. The nose interprets certain movements of the atoms as "smell," and our tongue perceives "taste" as soon as it is bombarded in suitable fashion by those invisible small corpuscles. Thus this world of ours, rich in beauty, glittering in the rays of the sun, is degraded to a cold monotonous host of atoms moving about

incessantly. And the scene of these movements is empty space into which all this has been immersed as if into a boundless gigantic receptacle.

It is true that Democritus, on account of this theory, was promptly declared mad by his compatriots. We men of modern times, however, know much better how to value his view of the universe in all its grandeur. For we clearly see revealed in him a tendency with which we are still familiar, the tendency to refer all processes of nature to motion as their final cause. And, after all, the individual sounds out of which Beethoven composed his *Ninth Symphony* are to us, too, nothing but vibrations of chords and air columns; we, too, do not doubt that a gaily assorted bunch of flowers in reality owes the abundance of its colours to the infinitely graduated rhythm of its rays of light, or that a warm body simply bears witness to the irregular changes of its molecular motions. In a word, we, too, accept a process as "explained" as soon as we know how to derive its origin from any processes of motion. For this reason we place at the head of our physics the theory of motion and of the forces causing them, *i.e.* mechanics, and since it was Newton, the above-mentioned English physicist, who laid the foundation of mechanics, we have also to regard this scientist as the originator of exact scientific physics.

The whole world process—the courses of the stars, all the physical and chemical phenomena, as well as all the happenings in the realm of atoms and electrons—such as I have described it to you in brief and rough outline, is finally reducible to motion. If we accept this view the ideas of space and time become essential constituents of our conception of the world, to which they appear absolutely indispensable. Stars and atoms are placed in space, and their motions interacting upon each other in many ways occur in time. To doubt their reality would appear as equivalent to a renunciation

of all knowledge. Without them all results obtained by scientific research would become meaningless and disappear beyond recovery.

And yet, in the most recent times, research has led to very strange results in this respect. "The" space and "the" time, pure and simple, are not to be mentioned any more. However unthinkable it may sound —we shall have to unlearn from the very beginning. The road towards this new knowledge will by no means be easy, but we shall have to tread it remembering Goethe's words in the "Westöstliche Divan":

> Till thou hast this truth possessed,
> "Die for higher Birth,"
> Thou art but a gloomy guest
> On this darksome earth.

II

THE FUNDAMENTAL LAWS OF GENERAL MECHANICS

IF you were ever taught physics at school you will remember mechanics with a shudder—the theory of motion and of forces with which instruction in physics is usually introduced. As a rule it appears to us as the driest and dreariest of all sciences, and we can hardly understand why physics, otherwise so interesting, should include such nasty territory. How you loved, for instance, to have shown to you, by experiments, the many marvels of electricity, and how you rejoiced in seeing the gay colour strip into which a simple glass prism would so quickly resolve the rays of the sun! But mechanics with its laws governing falling bodies and projectiles, its formulæ concerning pendulum and gyroscope, could hardly kindle your enthusiasm. And this was so probably for no other reason than that it was closely connected with mathematics, which in student song books has been derided again and again as the devil's art, and whose unpopularity with the general public has become a regular byword. And yet among all sciences mathematics occupies a privileged position for which it has often been envied. For which other science could compete with mathematics as to the certainty of results? When a philologist traces any word-structure back to its origin, he frequently may put forward a particular opinion in such a convincing manner that by the side of it other opinions can no

longer be held. But what is this compared with the unfailing certainty which the mathematician has a right to ascribe to his propositions without further ceremony ?

Indeed, this self-evidence of the mathematical system, this absolute impossibility of thinking that those propositions could be wrong, deserves most careful attention. Have you ever taken the trouble to consider why it should be so ? Comprehensive acquaintance with mathematics is by no means required for this purpose. The dim recollections still within you as remnants of former times will prove quite sufficient.

When you open any textbook on geometry you will, usually right at the beginning, find an explanation of those conceptions with which this science has to deal. That is to say, the definition of a point, a line, a surface, a body. Thereupon follow three fundamental statements, called axioms, usually formulated as follows :

1. Through two points in space one, and only one, straight line can be drawn.

2. The straight line is the shortest connection between two points.

3. Through a given point only one parallel line can be drawn to a given straight line.

These three statements do not need a proof. They are deeply rooted in our peculiar faculty of perception. Obviously we could only doubt their accuracy by completely renouncing our common sense. Later on we shall have to return once more to the reason underlying this fact. For the moment it may be sufficient to have the fact stated.

Now imagine any proposition in geometry, *e.g.* the well-known statement about the triangle, to the effect that the sum of its three angles equals exactly two right angles. Its accuracy is guaranteed by the so-called proof. But what does such a mathematical proof look like ? As a matter of fact it looks awfully simple. For it shows in a thoroughly logical, entirely unobjec-

tionable manner that the statement is certainly correct if certain other propositions previously dealt with are right. But those other propositions again depend, as their proofs show, on the validity of propositions dealt with still earlier. The validity of these propositions, too, is demonstrated in the manner described just now, and, continuing this procedure, one finally comes back to those simplest propositions of all that geometry knows, and these are the axioms enumerated above. In other words, by a more or less long chain of deductions you demonstrate that the particular proposition you are maintaining just then is a necessary deduction from those axioms. And since their accuracy is beyond doubt, the same must be true of that new proposition.

From this explanation you may readily imagine the immense importance which the axioms of geometry possess. They are the pillars on which the system of geometry is built. On them, and on them alone, rests its whole truth. If they were shaken the vast building supported by them would have to collapse. Unquestionably. No way out appears conceivable. But for the moment no danger is threatening them. Firmly embedded they stand, high above any, even the faintest doubt. And the building they carry towers proudly right up into the bright summits of the world of human thought.

Considering these facts, one need not be astonished to find that in other realms of science, too, repeated attempts have been made to secure the advantage of incontestable reliability of results by an application of mathematical methods of research. Just think, for example, of Spinoza's system of ethics, which was developed by entirely mathematical methods. Starting from a definition of the conceptions of which he is making use, he next gives definite axioms which cannot be proved, and which are said not to need a special proof. Then all further propositions are traced back to

these axioms by a logical chain of syllogisms, called proofs, exactly as in mathematics.

Physics, however, is without doubt the science which has adopted mathematical methods with the greatest success. And Newton, the English physicist whom we have mentioned before, may be called the real founder of mathematical physics. By referring mechanics to a strictly mathematical basis he produced the material for basing the whole of physics on mathematics. For, after all, mechanical processes were to be looked upon as the final causes of all physical phenomena.

Now, in order to make you acquainted with Newton's mechanics, let us start from an example taken from daily life. Imagine a man travelling on an uncovered truck of a goods train—let us say a truck used for the transport of felled tree trunks. While the train is moving forward in a straight line the man throws a leather ball high up in the air. What has he to do if he wants to catch his ball again? Has he to run forward, or backward, or stop where he is?

An answer to this question will hardly appear difficult to you. For during the time that the ball is moving upward and downward the train proceeds on its journey, and, unless it crawls exactly like a snail the ball, in all probability, will come down on a waggon much farther back, and our player might whistle for it. So you will come to the conclusion that one really ought to run backward in order to catch the ball.

The truth, however, is that in spite of the forward movement of the train, the ball, after having been thrown, will only return into the man's hand if he quietly stops in his place. However strange it may appear—you have known this fact from long experience. When a child, you used to throw the ball in the air running as fast as you could, and you never troubled your head about the fact that it always returned obediently into

your outstretched hands. In a closed carriage pulled by fast trotting horses, in the electric car, and in the fastest express—everywhere you notice the same thing. Familiarity, no doubt, has blunted your mind with regard to the miracle; you accept it without giving much thought to it, and the example chosen above only arrested your attention because it seemed a little out of the ordinary. The entirely open truck misled you, and made the result of the experiment described look so queer. With regard to the interior of a closed-in space, *e.g.* the compartment of a railway carriage, it appears quite natural to you that an object when thrown should not trouble about the motion of the whole. It just belongs to it, and consequently remains where it happens to be, exactly like the air, which, as you know, also remains quietly in the compartment, at least as long as the windows are kept closed. But on an open truck? Over which the air rushes along as if a gale were blowing? There, too, the same rule is to hold good, even if the ball is thrown up a distance of many metres? This really sounds very remarkable, and only actual observation is likely to convince you fully of the accuracy of our statement.

Consider for a moment the following: if the ball, after having been thrown, is really to return into your hands, even if you on your truck are moving in a straight line, then the ball evidently has to remain exactly over your head all the time, just in the same way as it does when you throw it while standing on a truck at rest. Looking upward, therefore, you will see the ball hovering over you all the time, at first rising, its motion growing slower and slower, afterwards descending again with ever-increasing velocity. Thus it will appear to you and your fellow-travellers. On the other hand, an observer at rest whom you pass on your journey will see the path of the ball quite differently. Because for him the ball carries out two motions simultaneously! One

of them upward and downward, just as for you. But apart from this motion—and simultaneously with it—for the observer standing outside, the ball travels in the direction of your journey. And for him these two motions are combined into one which, as experience teaches us, appears to take place in a curved line, resembling a parabola.

This fact enables you to see quite clearly what is really so strange in the whole affair. The ball in the air, thrown from the moving truck, participates all the time in the motion of the truck. As long as it was resting on the truck, this is obvious. But even after it has been separated from the truck, as a result of the throw, this throwing motion cannot prevent it from continuing its original motion.

With these words we have pronounced the contents of the first axiom of Newton's mechanics. In scientific language it is usually expressed in the following terms: every body continues in a state of rest or of uniform rectilinear motion as long as external causes do not prevent it from continuing in its state.

This statement, in reality, is based on experience. The fact that a body at rest remains at rest as long as it is undisturbed is so obvious to all of us that we need not waste any further words on it. But the second part of our statement appears to you distinctly more extraordinary. According to it a body in motion, too, is said not to terminate its motion, assuming only that its motion is rectilinear and uniform. You know, of course, what is meant by rectilinear motion? As to uniformity, we call a motion uniform when its velocity remains the same all the time; in other words, when it is never either accelerated nor retarded. Of such a motion our axiom states that it continues unchanged throughout all time.

At first sight this certainly appears strange to you, and it seems that we have no right to say that this fact

is in accordance with our experience. One would rather say that every motion left to itself after a period more or less short will come to an end and pass into a state of complete rest. The pendulum of our clock stops when we forget to wind the clock up, and the billiard ball does not roll along eternally unless we continuously impart new impulses to it. Evidently a glaring contradiction to our statement. Don't you think so?

And yet, just look at it a little more carefully. We stated most emphatically that the axiom could only claim validity if there were complete freedom from all constraining influences. This freedom, however, is by no means guaranteed in our instances. Because hitherto we have completely omitted two important circumstances. First of all, the pendulum of the clock as well as the billiard ball carry out their motions through the air; and secondly, both have to struggle with friction—the pendulum at its point of suspension, the ball in its gliding motion along the cloth. But air and friction are resistances to motion, and consequently there is no freedom from constraining influences which our axiom, to be valid, postulates as a necessary and indispensable supposition. Consequently we have no right to talk of contradiction; we may rather see in those two instances a welcome confirmation of our axiom.

A *confirmation*, no doubt; this is not to be denied. But evidently you would wish for more; you would like to see it strictly *demonstrated* by experiment that our statement is correct. But you will have to consider that we are dealing with an *axiom*. And just as we pointed out the impossibility of proof with regard to the axioms of mathematics, in the same way you will readily understand that here, too, we cannot produce a regular proof. Because on this earth we can never completely get rid of all resistances to motion. We can only arrive at more or less rough approximations by perfecting more and more the experimental arrange-

ments in our laboratories. And from those approximations we deduce, in a bold generalisation beyond everyday experience, that these disturbing influences must be unimportant and incidental, and foreign to the *ideal* process of motion as such. This ideal process of motion never reveals itself to us in its pure form. It combines with foreign influences of a different character into a uniform group of phenomena which in nature faces us as a whole, and from which we have to pick painfully —by a sort of sifting process—what is really essential. Thus our axiom in its final aspect becomes a purely abstract fact of knowledge, for the production of which observation merely supplied the occasion.

If the axioms of geometry and mechanics thus show a certain similarity which we acknowledged by applying the same terms to both of them, nevertheless you cannot overlook an important difference between them. Whereas the axioms of geometry express the specific peculiarities of empty space, the axioms of mechanics are restricted to the material contents of this space. Consequently mathematics appear to be bound up inseparably with the existence of space, physics, on the other hand, with the existence of a material world which, according to our experience, occupies that space in the form of celestial bodies and chemical atoms. Now, as a matter of course, you could easily imagine that everything in our universe were non-existent ; space " in itself," however, this boundless receptacle of the world, resists all attempts at thinking it away. For this reason the mathematical method of investigation, the peculiarities of which we have just now described, as applied to geometry, leads straight away to truths which we dare not dispute or doubt. In mechanics, on the other hand, the same method only produces results, the accuracy of which, although probable to a very high degree, has in every case to be verified by experience. If experience should prove them to be wrong this could not be attributed

to the method but to the behaviour of the bodies, to certain qualities of matter which had remained unknown to us till then, and which we consequently had not taken into consideration in our investigations. In other words, phenomena of nature do not occur to us as geometrical necessities, but as accidents of material peculiarity, the complete understanding of which is to be regarded as the highest aim of physical science.

But now let us return to our discussion of the fundamental laws of mechanics. The first of these axioms has very suitably been called the principle of inertia. It was so called because it expresses the inertia, in a certain sense indwelling in bodies, which prevents them from changing their state of motion, no matter whether they are at rest or in uniform rectilinear motion. Thus if we—in contradiction to the statement of the principle of inertia—see a body at rest starting to move, or see a body already in motion suddenly stop, or when a body moving uniformly in a straight line deviates from its rectilinear course, or accelerates or retards its velocity—we shall, in all such cases, have to conclude at once that there is a cause bringing about the deviations which we observe. However different these causes may be in character, the physicist calls them by the common name of forces. Thus the word force denotes nothing but the cause of any change of motion, and as the measure of the force we shall have to regard the amount of change of motion obtainable by it. But in what way can the changes of motion appearing in a body which obeys the law of inertia show themselves? Just now we enumerated all possible changes, and from this enumeration it is easy enough to see that, in the end, they all amount to a change of direction and velocity. When velocity changes, motion is either retarded or accelerated. And a mathematician could easily demonstrate to you that a change of direction in a moving body means nothing but the appearance of an accelera-

tion whose direction differs by a definite angle from the original direction of the motion. Thus every change of motion may be called briefly an acceleration or retardation. The greater the acceleration brought about by a particular force the greater will be the force in question. But, in addition, the quantity of the moving mass plays a part in the affair. For if you wish to impart to a greater mass the same acceleration as to a smaller one, you naturally will have to use a greater force than in the case of a smaller mass. Imagine you were trying to push away a big lump of iron and a billiard ball, and you will soon know what I mean. Thus a force is measured by the quantity of a mass, and the acceleration produced by the force in this mass, and it is equal to the product of these two factors. This statement is the second axiom of Newton's mechanics.

You will, no doubt, be astonished that we have discussed these laws of forces in such a brief manner. We may do so without any qualms of conscience, because later on we shall only make very little use of them. The same applies to the third fundamental law of mechanics, which we will only explain in a few words for the sake of completeness. When a ball is being fired from a cannon, this ball is thrown from the state of rest into one of motion of considerable velocity, by the force of the compressed powder gases. Simultaneously the cannon, although at rest till then, will experience the effect of a force working in the opposite direction and showing itself in a backward motion of the cannon. Now, if you measure the velocity of the flying ball and multiply it by the mass of the ball, you will obtain exactly the same result as when you multiply the velocity of the moving cannon by the mass of the cannon. Since similar observations apply to all forces, Newton expressed, in his third axiom, the fact that the effect of each force always appears to be accompanied by an equal effect in the opposite direction.

General mechanics, therefore, has to begin its considerations with these three fundamental laws—the principle of inertia, the law of forces, and the so-called law of equality between action and reaction. They are the basis of the system of mechanics, and all other mechanical laws of nature can be derived from them. But not only the mechanical laws of nature. For as you have seen from our description of the conception of the world adopted by modern physics, all phenomena in nature are said to be finally nothing but processes of motion of some kind or another. In this way the axioms of general mechanics become the basis of the whole system of physics. Very characteristic in this respect is the well-known statement made by Huygens, according to which in real science one could only understand the cause of all effects by adopting methods of mechanics, and this would have to be done without reserve unless we were prepared at once to renounce all hope of ever understanding anything in physics.

III

THE SYSTEM OF CO-ORDINATES

PHYSICS, as we have repeatedly stated, would like to reduce all phenomena in nature to processes of motion as their final causes. It is understood, as a matter of course, that those processes of motion are taking place in empty space. Here you will feel inclined to ask how, in empty space, the presence of motion can be detected at all? You thus raise a question which is extremely difficult to answer, and which requires a great deal of thought. Its final answer will play an important part in our further considerations.

However, let us first ask a much simpler question: How are we able to declare with certainty that any object, let us say in our room, actually is in motion? To give an example that clearly illustrates this point: How do you know that a ball is moving on our writing-desk? The answer seems obvious. For, in case the ball is moving, it has, in the course of time, to change its place on the desk. So much is certain without a doubt. Equally certain is, further, the fact that such a change of place executed by the ball can be ascertained by watching, at various moments, its position with regard to fixed (*i.e.* at rest) objects on our writing-desk. If, therefore, at a given moment, the ball passes our inkstand, but soon after is to be found near the ash-tray, nobody, I imagine, will dispute our assertion that the ball has moved. For since we know with certainty that during this time the inkstand as well as the ash-tray have

remained in their positions without moving, the ball has changed its place, and thereby executed a motion.

The affair is somewhat more difficult if there is neither inkstand nor ash-tray, nor any other fixed object on our writing-desk. But in this case, too, we shall find a way out after brief consideration. Let us call, as shown in Fig. 1, the four corners of the writing-desk A, B, C, and D.

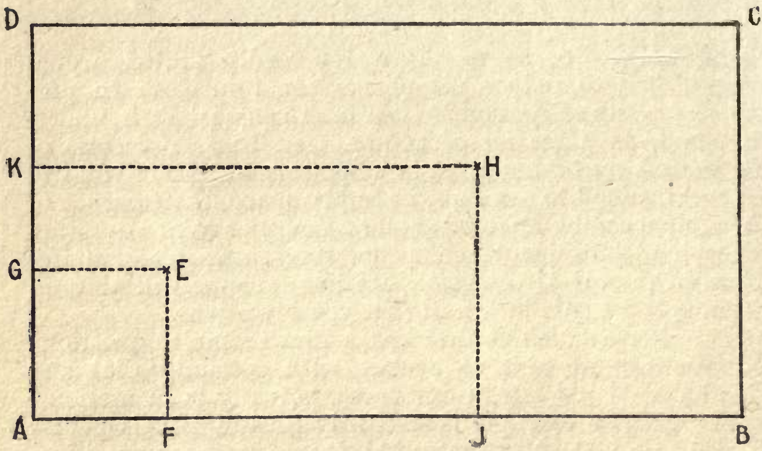

FIG. 1.

We can then easily find out the place at which our ball is situated at any particular moment by ascertaining the distance between its centre and the two sides of the desk, AB and AD. By distance between a point and a straight line, we mean, as you know, the length of a perpendicular line drawn from that point to the straight line. Thus, if E in our figure denotes the present position of the ball's centre, EF is its distance from the straight line AB, and EG its distance from the second straight line AD Soon after the ball's centre is

found in point H; now the lines HJ and HK are its distances from the two straight lines. Now, if you compare the values, expressed, let us say, in centimetres found first for EF and EG, and then for HJ and HK, they will turn out to be different. Thereby it becomes certain at once that the ball's centre has changed its place, and that the ball has executed a motion. With the help of the diagram you will straightaway see the accuracy of our conclusions.

It looks, therefore, as if the task which we had placed before us can be performed in a satisfactory manner. Applying the measuring process just described, we shall, I dare say, always be able to prove indisputably that the ball is moving across our writing-desk. But stop, there still is a hitch of some importance in the matter. Just imagine for a second we had the ball held in position by somebody, while we were pushing the writing-desk along under it. Should we in this case, too, not feel constrained by our measurements of distances to draw the same conclusion that the ball had moved? No doubt this would have to happen. For so much you see at once, that in this case, too, the position of the ball on the writing-desk will have changed; our measurements, therefore, are bound to lead us to the assertion that a motion of the ball has taken place. Yet, in reality, it was the table that moved while the ball remained at rest. However strange it may seem, we should have to draw that conclusion. But as it is a wrong one we must, of course, try to avoid it. And how could this be done?

It would be simple enough, for after a little reflection we easily see the reason why we went wrong in our conclusion. At first we emphatically stated we were going to ascertain the ball's change of place by comparing its position with objects at rest, while, in the end, we used the edges of our writing-desk as such. They, however, were not at rest; they were, with the

rest of the writing-desk, being moved through the room! Consequently we have no right whatever to use them for ascertaining the motion of any other bodies. We ought rather to look round again for objects at rest, in order to reach our aim with certainty. In our room such objects at rest are presented to us most conveniently by the walls. If we, then, while the desk is being moved, ascertain the distances of the ball from these walls at various moments we naturally shall always find the same values. Consequently the ball has been at rest. The corners of our desk-top, on the other hand, will show, in the course of time, different distances from the walls; so they are always in different places of the room, and thereby make it certain that the table is moving. In this way we avoid, without difficulty, the deception to which we were at first in danger of succumbing.

In spite of all this you will hardly be able to rid yourself of a certain feeling of uneasiness. How are we likely to fare with our real task, the question as to how motions in empty space are to be ascertained with greatest certainty, if we already have to face such great difficulties? Still, do not let us despair at the beginning of the road! As the basis of our conception of the world we have got, as you know, the vast extending space, in a sense the large, infinitely large, empty "box," inside which everything that happens in nature takes place, and which we have an unquestionable right to imagine as being "absolutely at rest." For all our present, as well as our future knowledge can never reach beyond the boundaries of this space, should such boundaries exist, and without fear of contradiction we may call any question as to a possible "motion" of this "space in itself" altogether meaningless. We, too, with a quiet conscience, may therefore regard space as that empty receptacle in which, nearly twenty-five centuries ago, Democritus made his atoms rush about in mad processions, deceiving silly human senses by the

illusion of a world full of sun, colour, and form. And since our latest considerations have shown us the way, we shall now see without difficulty that it is possible after all to determine position. We imagine built in that empty space at rest a structure—shown in diagram 2 in perspective—

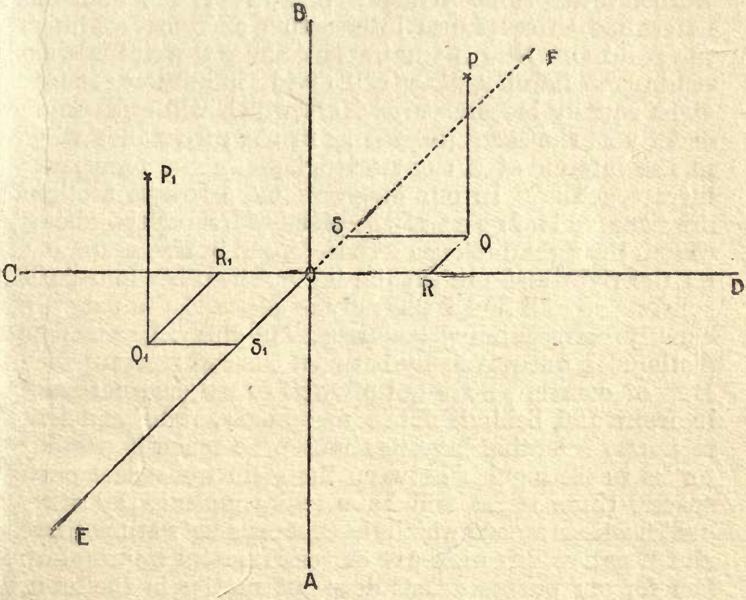

FIG. 2.

consisting of mathematical plane surfaces possessing only length and breadth, but no thickness. In this diagram AB and CD are two straight lines situated in the plane of the paper, and intersecting each other at right angles in point O. EF, on the other hand, is to be imagined as a straight line in point O, perpendicular to the plane of the paper which it pierces, so to speak,

forming right angles with the lines AB and CD. And in the same way as the plane of the paper is determined by the two straight lines, AB and CD, the two straight lines, AB and EF, as well as CD and EF, determine two new planes, both of them being perpendicular both to the plane of the paper and to each other. In other words, those three straight lines, AB, CD, and EF, determine three planes intersecting at right angles in the point O. Now if you imagine those three planes extending to infinity, they will divide infinite space into eight equally infinite part-spaces which will meet at O, exactly in the same manner as in the corner of a room in the interior of a two-storied house eight rooms will meet together. In our diagram four of these sections are situated *in front* of the plane of the paper—above, one on the right and one on the left, and below, again, one on the right and one on the left. The other four part-spaces you will find *behind* the plane of the paper in exactly corresponding positions. In this way you can distinguish unequivocally between the eight part-spaces. But, of course, we are not allowed to use such terms as in front and behind, above and below, right and left, in purely scientific investigations. To make it possible for us to distinguish between the eight individual part-spaces, those terms will have to be replaced by other qualifications among which the contrasts between positive and negative direction are of fundamental importance. But for our purposes that does not matter in the least. For us it is sufficient to know that it is possible to distinguish these part-spaces from each other unequivocally by definite statements. And once you have grasped this point—no matter by which road you arrived at it—you will, without further trouble, at once understand how we are able to describe the position of any given point in infinite space. First of all, we shall have to state in which part-space of our structure the point is to be found. Thus, for instance, point P in diagram 2 is

situated behind the plane of the paper in the upper right-hand part-space. Its exact position within this part-space is determined by the three lines, PQ, QR, and QS, by which its distances from the three limiting planes of that part-space are measured. On the other hand, point P_1 is situated in front of the plane of the paper in the upper left section, and the lines P_1Q_1, Q_1R_1, and Q_1S_1 again determine, in an analogous manner, the exact position of point P_1. So, if at one time we find a particular body at the point P, and at another time at the point P_1, we have, with certainty, established the fact that the particular body has carried out a motion in empty space.

This peculiar structure composed of planes, with which you have just become acquainted, is briefly called by the mathematician a system of co-ordinates in space. By such a system of co-ordinates, therefore, space is divided into eight part-spaces which meet in the point O, the so-called origin of the system. The distances by which a given point is separated from the three limiting planes of its section are called the co-ordinates of the point in question with regard to the system of co-ordinates chosen.

IV

ABSOLUTE SPACE

NOW let us glance once more, in all brevity, at the results obtained so far from our considerations. To theoretical physics we had assigned the task of making all events in nature intelligible to us by processes of motion in empty space. In order to establish and describe motions in empty space we need a system of co-ordinates, the origin of which is firmly anchored somewhere in the space at rest. It is practically a matter of indifference which point we choose for this purpose. We have a free choice among the infinite number of points which, in their totality, constitute infinite space. But one thing is absolutely certain—after having once selected a definite point, and having made it the origin of a system of co-ordinates at rest, we evidently shall have to retain this point unchanged for all further investigations if we wish our descriptions of motions to have any sense at all. Analogous considerations apply to the three planes of our system. With regard to them, too, it is in itself immaterial which directions we give to their positions in space as long as they intersect each other at right angles in the point chosen by us. As soon, however, as we have decided upon a definite position, we must not change it in any way afterwards.

These being the facts of the case it will be highly desirable in future to leave the conception of space alone as far as possible. For once you begin seriously

to think about it you will soon come upon insurmountable difficulties. On the other hand, in our system of co-ordinates at rest we possess a suitable means of representing empty space to ourselves as something almost tangible. Its three plane surfaces partition off the whole of empty space without leaving a gap anywhere, and, by referring it to the three co-ordinates, any, even the remotest point in space, is unequivocally described as to its position. Thus we are led to identify space at rest with our system of co-ordinates at rest in this space, and to regard it as the task of physics to describe the visible and invisible motions of all matter existing in nature, with reference to that system of co-ordinates.

Still, the solution of this task is beset with considerable difficulties. For we have to take it for granted, as a matter of course, that we have at our disposal some such point in space at rest, and are able to attach to it our system of co-ordinates at rest. But where is such a point to be found? In the continuously moving universe of the stars a point at rest of which old Archimedes once was dreaming to lift the world out of its hinges by means of it? Where can we get hold of the infinite space at rest, we who are compelled to participate in the daily rotation of mother earth about her axis? We who can only look on passively when year after year the earth flings us round the sun in its gigantic orbit? We who have to read in our books, daily and hourly, what a proud triumph it was for astronomy to find out that the burning ball of the sun, together with its whole system of planets, is rushing through space on an unknown path? Where do we find the space at rest?

The answer to that question is closely connected with those considerations which, in the course of time, have led us to deny to the earth that position at rest in the centre of the universe which everywhere, it seems,

primitive human thought in a low state of culture was fond of assigning to her.

The pendulum is a physical apparatus well known to you no doubt. In its simplest form it usually consists of a small ball hung on a thin thread. As a rule the ball, by its weight, pulls the thread firmly downward, and, in accordance with the law of inertia, the pendulum will continue in this state of rest as long as it is not exposed to any effects of forces. But if you lift the ball up sideways, taking it out of its state of equilibrium, and then let it go without pushing it, the pendulum will swing to and fro. From its three fundamental laws scientific mechanics is able to derive, by mathematical methods, the laws governing these oscillations. The law of inertia, in particular, leads up to the postulate that the oscillations of the pendulum do not change their direction, but have, all of them, to take place continuously in the same plane. Actual observation, however, led to a very different result. For, in 1852, when the French physicist, Foucault, in the cupola of the Pantheon at Paris attached a ball weighing about thirty kilogrammes to a steel wire nearly seventy metres long, and made the whole thing swing as a pendulum, after a few hours it became apparent that the direction of the oscillations was rotating quite distinctly.

The state of things, therefore, was this—the experiment which Foucault performed with the pendulum contradicted the fundamental laws of mechanics. Or, in a more explicit and scientific way of speaking, if we make a pendulum oscillate we need, as you know from our previous explanations, a system of co-ordinates to describe its motion. In Foucault's arrangement of the experiment the oscillations of the pendulum were, naturally, judged with reference to the surface of the earth, *i.e.* with reference to a system of co-ordinates rigidly attached to the earth. If, in such a system of co-ordinates, we determine the co-ordinates of the centre

of the pendulum's ball at as many moments as possible during a single oscillation, we obtain a consecutive number of positions which together form an arc of a circle. You will notice this immediately on looking at Figure 3*a*. The direction of the oscillations which we are investigating is marked by the straight line AB. If the motion of the pendulum actually obeyed the rules which follow from the fundamental laws of mechanics,

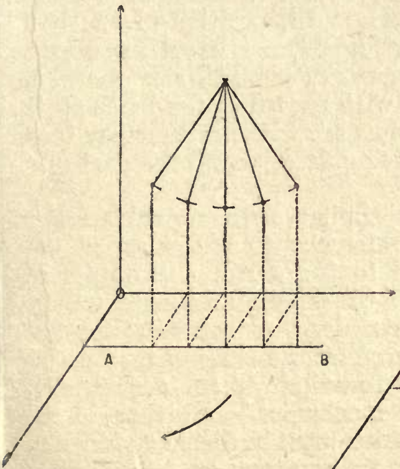

FIG. 3*a*.

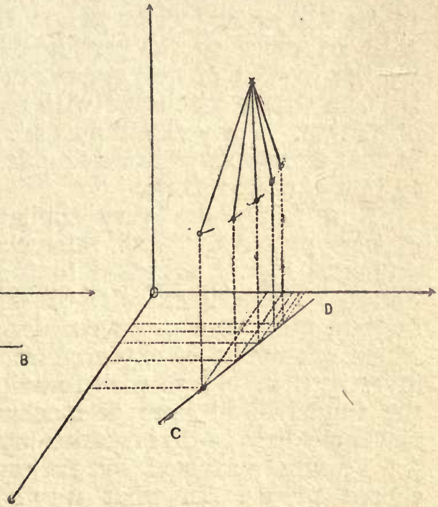

FIG. 3*b*.

a repetition of the measurements just described ought to produce exactly the same results after the lapse of a certain time—let us say an hour. Consequently, if we made a drawing of it we again ought to get our diagram 3*a*. As a matter of fact, however, we obtain an entirely different state of things, as illustrated by diagram 3*b*. From it you will clearly see that the course followed by the ball of the pendulum has remained an arc as before.

But its direction has turned from the original position AB into a new position CD in flagrant contradiction to the fundamental laws of mechanics.

The difficulty in which we have got entangled is unmistakable. But how are we to remove it?

Just call back to your mind the example in which we proposed to have our writing-desk pushed along under a ball held in position. At that time we saw that in such a case we should be bound to ascribe a motion to the ball as long as we judged the position of the ball with reference to the writing-desk. In other words, by the motion of our system of co-ordinates we were led to a wrong conclusion with regard to the ball's state of motion. For you will now see without difficulty that in that experiment the writing-desk played the part of a system of co-ordinates.

Now things are very similar with regard to our present problem. With reference to a system of co-ordinates rigidly attached to the earth, the results of Foucault's experiment with the pendulum contradict the fundamental laws of mechanics. A rotation appears which was not to be expected on the basis of theoretical investigation. If we wish to adhere to the accuracy of the fundamental laws of mechanics—and thus of all mechanical laws—that rotation must be due to an illusion. An illusion produced by the system of co-ordinates chosen by us. In what way? Simply because our system of co-ordinates rotated during the hour which elapsed between our two measurements. And it must have rotated in the direction of the arrow which we have drawn in diagram 3*a*. A brief consideration will show you that, assuming this rotation, we actually obtain the picture represented in diagram 3*b*.

But what is the simple meaning of our highly learned explanation? No other than this, that our earth is carrying out a rotary motion about an axis passing through her north and south poles. In other words,

by Foucault's experiment with the pendulum the daily axial rotation of the earth is demonstrated.

There are other experiments of a different character which supply further proof for the rotation of the earth. But I will not bore you by describing them in detail. Otherwise you might take me for a regular schoolmaster, and take to your heels at once, panic-stricken by the amount of learned ballast with which, in that case, I should have mercilessly to burden you. And, after all, I can let you off with a clear conscience, because the conclusions in all cases are exactly the same. For all those experiments contradict the fundamental laws of mechanics as long as you refer them to a system of co-ordinates rigidly attached to the earth. And, in most cases, this contradiction disappears as soon as one takes the axial rotation of the earth into consideration. In most cases, I say; but not always. For certain phenomena show a distinct deviation from the expectations gained theoretically, even if we take the axial rotation of the earth into consideration. But this difficulty, too, is easily removed. Their harmony with the postulates of theory is at once re-established if we ascribe to the earth an annual revolution round the sun, that is to say, if we attach our system of co-ordinates no longer to the earth, but to the sun. The rotation of the earth, therefore, and her revolution round the sun, prevent the unrestricted validity of the mechanical laws of nature on the surface of the earth, but to such a small degree that the deviations need not be considered as far as our daily life is concerned. Their influence can only be shown by scientific observations carried out with greatest care.

Thus we are led to attach to the sun the system of co-ordinates to which we mean to refer the results of our physical investigations. But we have no right to attach it definitely to the sun either. For most delicate astronomical measurements have led to the surprising result

that a system of co-ordinates attached to the sun does not guarantee a strict validity of our fundamental laws either. In other words, the sun, too, is in motion; the sun, too, is rushing through space on a tremendously vast, boldly curved path. And thereby the physicist is compelled to move his system of co-ordinates once more.

But where is he to put it now? In perfect despair you are asking this question, and no suitable answer occurs to you.

Here, however, we find our way back to those lines of thought which we have left since we discussed Foucault's experiment with the pendulum. They, too, ended in a great and difficult question, the question as to where a point at rest is to be found in the universe, in which we could firmly fix the origin of the system of co-ordinates at rest which was to represent the space at rest. We are now able to give a satisfactory answer to both questions, the previous one as well as the one just referred to.

First of all this much is certain—in order to describe natural phenomena we can only use a system of co-ordinates really at rest. For all systems of co-ordinates which are moving in any way make the fundamental laws of mechanics appear invalid. Therefore neither earth, nor sun, nor any other celestial body known to us comes into consideration, as the bearer of the system of co-ordinates in question; only a point at rest in the universe may be adopted as the origin of this system. But now we know, too, how to find this point. To test our statement we only have to fix a system of co-ordinates in any point of space, and then refer all our observations to this system of co-ordinates. What this means will now be obvious to you. As long afterwards as the results are in contradiction to the fundamental laws of mechanics we shall have to choose ever new points, *i.e.* introduce ever new systems of co-ordinates. Until finally—purely by chance!—we shall arrive at a system

which guarantees the validity of the fundamental laws of mechanics, in which, therefore, all consequences derived from theoretical mathematics can be confirmed with strict accuracy by experimental tests with regard to all mechanical processes imaginable.

We have hitherto purposely always spoken of the system of co-ordinates at rest, not of space at rest, which, as you know, is identical with it. We will now make an exception in order to summarise, clearly and briefly, the results obtained so far. By absolute space at rest we mean that space in which all laws of nature, in particular the fundamental laws of mechanics, are fulfilled with absolute accuracy.

And now our whole problem seems to be solved to our complete satisfaction. Possibly that testing of systems of co-ordinates, mentioned above, will not be to our liking at first, but you may calmly leave it to physicists. Let it be sufficient for you to recognise that honest endeavour is bound to lead to the goal, *i.e.* to a system of co-ordinates to which we can refer all phenomena in nature without the constant fear of being misled by fallacies.

V

THE PRINCIPLE OF RELATIVITY IN CLASSICAL MECHANICS

YOU know the scene in Goethe's *Faust* where, after a refreshing walk on the evening of Easter Sunday, Faust, in deep meditation, begins to translate the Gospel of St. John into his beloved German. And you know, further, what difficulties he has in finding a suitable translation for the first words of the original text.

We have had to go through similar experiences in our endeavours to get hold of a space absolutely at rest. Again and again we found ourselves disappointed in our hopes of having reached the longed-for goal, and again and again we renewed our efforts. Until, in the end, the "spirit" came to our help too, so that we, too, "suddenly saw a way out," and recognised clearly and distinctly the true character of the space at rest.

Our conception of the world, which we described at the beginning, receives thereby a powerful support. For you remember the important part played in it by empty space. In the truest sense of the word it formed the basis—being the scene of the infinitely multifarious processes of motion which constitute the essence of all events in nature.

In spite of all this I cannot help causing you yet another great disappointment. The greatest of all. A disappointment from which no deliverance will be possible, except by sacrificing, no matter at what cost,

ideas familiar to you of old. Subsequently, however, you will reach heights of knowledge hitherto undreamt of. Still, the road leading to that goal is long ; prepare yourself to walk it with courage.

Let us assume that we had actually discovered the space at rest, and that we had got a system of co-ordinates with reference to which the phenomena of nature take place in fullest agreement with the fundamental laws of mechanics. Now let a carriage travel through that space without friction, and without any other resistances. Let its motion take place on an absolutely straight course, with a permanently unchanged velocity of, let us say, ten metres per second. Since this carriage executes a uniform rectilinear motion, according to the law of inertia, it will never discontinue its motion. Never, because in this absolute space the laws of nature hold with absolute accuracy as we know. Let a physicist be in that carriage, having a completely fitted out laboratory at his disposal, equipped with all instruments required for taking the most accurate scientific measurements. Now, in this space at rest, let us set a ball in motion, exactly in the same direction in which that carriage is travelling. This ball too, therefore, is moving in a straight line ; we take care that, in addition, it moves uniformly, and we impart to it a velocity of, let us say, one metre per second. With reference to our system of co-ordinates at rest, we shall arrive, then, at the following observations : in every single second the ball advances on its rectilinear course by exactly one metre. It is a matter of complete indifference whether I make my respective measurements to-day or after the lapse of any period of time. I shall always obtain the same result. There is nothing strange in this, it is rather an obvious consequence following from the fact that I am in space absolutely at rest, and consequently with reference to my system of co-ordinates the motion of the ball has to obey the law of inertia. I therefore

summarise my observations in this brief statement: the ball, with reference to my system of co-ordinates, is moving in a uniform rectilinear manner, strictly obeying the law of inertia.

Now let the physicist in the carriage observe the motion of the ball by measuring it continuously. What is the result he will arrive at?

A simple consideration will supply the answer. Let us assume him to be starting with his measurements just at the moment when the ball, moving outside the carriage, is exactly opposite him. For this position he determines the co-ordinates of the centre of the ball with reference to his system of co-ordinates, and then he waits exactly a second before repeating the measurements. During this second the ball, as compared with our system at rest, advances by one metre, while the carriage of the physicist is advancing by ten metres during the same time. So the system of co-ordinates of our physicist, rigidly attached to the carriage, outpaces the ball by nine metres, and when he, after the lapse of a second, determines the co-ordinates of the ball's centre with reference to *his* system, he will find that, compared with its original place, its position has now been shifted backward by nine metres. The physicist, consequently, will come to the following conclusion: with reference to his system the ball is moving backward in a straight line with a velocity of nine metres. You will understand what we mean by the term "backward." In itself it does not mean anything, and a mathematician would express the fact quite differently. But we are not mathematicians, and have no wish to be. Thus we cannot help making use of that inadequate term in order to denote that our physicist will observe a motion of the ball whose direction is exactly contrary to the one appearing to us, judged from our system, which is absolutely at rest.

Our physicist will now wait another second before

determining once more the co-ordinates of the ball's centre with reference to *his* system. You can easily foresee his result. For during that second, with reference to *our* system at rest, both ball and carriage have advanced in their original direction with their original velocity. The ball by one metre, the carriage by ten. Consequently the system of co-ordinates attached to the carriage has again outpaced the ball by nine metres, or, the ball has been left behind by the system on the carriage by nine metres. Our physicist, therefore, will find the new position of the ball at a point of *his* system which, compared with the one found just before, appears shifted back by nine metres. Consequently, reasoning in an entirely consistent fashion he will come to the conclusion, during the second second, too, with reference to *his* system of co-ordinates, the ball has moved backward in a straight line with a velocity of nine metres. And however frequently he may repeat his measurements—he will invariably arrive at the same result. For as compared with *his* system of co-ordinates the ball is constantly left behind by exactly nine metres per second, and its motion always proceeds exactly in the same straight line.

The final opinion of that physicist will therefore be like this: the ball, with reference to *his* system of co-ordinates, is moving uniformly in a straight line, strictly obeying the law of inertia. And as strictest obeyance of the fundamental laws of mechanics is said to be the characteristic of the system of co-ordinates absolutely at rest, he will quite consistently declare *his* system to be absolutely at rest.

There we are face to face with a nice mess. We, observing from our space at rest, see with absolute certainty a carriage moving through our space. And the physicist in this carriage is gaily to make the assertion that his system is absolutely at rest, that *his*

carriage consequently is at rest with respect to our system as well!

But since the man is a sensible physicist, I dare say he will listen to reason, if we get into communication with him. We shall point out to him in all politeness that he evidently is in the wrong. For it is downright nonsense for him to go on declaring his carriage to be at rest while it is actually moving. We shall explain to him that we in our space, which is absolutely at rest, may be permitted to express a reliable opinion about the state of his motion, and he will have to submit to our statement.

But how is that man likely to reply to us? In all probability in the following manner: That he didn't see the slightest reason why he should withdraw his original opinion. For with reference to his system the fundamental laws of mechanics possessed absolute validity, consequently his system was the one truly at rest. We, on the other hand, with reference to his system, were in a state of uniform rectilinear motion, moving backward by ten metres per second. And if then, once more, we were to point out to him the absolute character of our system he, by way of return, would ask the question how we had got to know about the absolute character of our system? There would be no other answer left to us but that with regard to our system, too, the laws of nature possessed absolute validity. And the whole dispute would have to end without decision.

Of course there would still be a possibility of bringing the affair to a profitable conclusion. For up to now we have only been working with the first fundamental law of general mechanics, leaving the other two severely alone. So we will now let a force of some kind act upon the ball in motion, a force acting in the direction of the ball's original motion. The real size of this force we shall find, according to the second fundamental law of

mechanics, by multiplying the mass of the ball by the degree of acceleration imparted to the ball under the influence of the force. The mass of the ball will evidently be found to be the same in our system at rest as it is in the carriage in which the physicist is moving. But what about the acceleration? By acceleration we mean the rate of increase of velocity received by the ball in one second. In a space which is absolutely at rest this increase of velocity must be exactly the same in each second. This is so because with regard to such a system of co-ordinates at rest the fundamental laws of mechanics are strictly fulfilled, and consequently we are bound to find exactly the same value for one and the same force at any instant of time. Hence, with regard to our system, we shall observe how, under the influence of the force, the motion of the ball grows faster and faster, and if, for example, the increase of velocity during the first second amounts to five metres, we shall find the same acceleration at any subsequent time.

Now, what is the physicist travelling in the carriage likely to observe? Let us, for simplicity's sake, assume that the force again begins to act just at the moment when the ball is opposite the physicist. During the first second the carriage with reference to *our* system advances by ten metres. The ball, too, advances, not only by one metre, owing to its original velocity, but, apart from this, by another five metres, owing to the effect of the force, altogether, therefore, by six metres, so that, with regard to the carriage, it is left behind by four metres. Thus the physicist, by the end of the first second, will find the velocity of the ball to be four metres, *i.e.* five metres *less* than it would be without the effect of the force. By the end of the second second the carriage has proceeded by another ten metres, with regard to *our* system; but the ball has, first of all, advanced six metres under the influence of its previous velocity, and then another five metres under the con-

tinued effect of the force, *i.e.* eleven metres altogether. Consequently the ball now is ahead of the physicist's system by one metre; as a result, the physicist, with respect to his system, will form the following opinion about the situation: the ball has come under the influence of a force acting in a direction exactly opposite to the original direction of the ball's motion. Thus the motion of the ball had first been retarded; it moved backward more slowly, came to rest for an instant, and then moved forward in a straight line. In the course of time this forward motion grows faster and faster, the velocity increasing by exactly five metres per second. In order to see this you only have to call back, once or several times if necessary, the situation as we were considering it just now. The final result will be that the physicist in his moving carriage, just like us in our system at rest, will come to the conclusion that the moving ball is acted upon by a force acting in a straight line forward and imparting to it an increase of velocity of five metres per second. In other words, from both systems the same opinion will be formed as to the magnitude and direction of the force acting upon the ball, so that with regard to the second fundamental law, too, both systems of reference have to be considered as fully equivalent. Therefore both, we and the physicist, have a right to assert with deepest conviction that we are in a space absolutely at rest.

And a consideration with respect to the third fundamental law of mechanics leads to exactly the same result. The decision we are out to find cannot be reached by its help either. So we are at the end of our tether. For if the three fundamental laws of mechanics possess strict validity with regard to both systems, the one at rest and the one in motion, then, automatically, the same will apply to all laws of nature. Or, in other words, all mechanical phenomena in nature take place with reference to the

system of co-ordinates at rest in exactly the same manner as they do with reference to another system of co-ordinates which is in uniform rectilinear motion with regard to it.

If this means anything at all it means that with regard to mechanical processes we have no longer a right to speak of a "space absolutely at rest." For we have seen that, in principle, it is impossible to establish its existence by mechanical experiments. The system of co-ordinates "at rest" is in no way whatever distinguished from the infinite number of all systems of co-ordinates that are "in uniform rectilinear motion." The conceptions of "absolute rest," as well as "absolute uniform rectilinear motion," become entirely meaningless; they cannot be detected, nor explained. "Rest" and "uniform rectilinear motion" are relative conceptions, *i.e.* conceptions which only have a definite meaning with reference to some system of co-ordinates.

The knowledge of this fact is by no means new. On the contrary, it has been well known since the foundation of scientific mechanics was laid by Galilei and Newton, and for this reason to-day we briefly denote it as "the principle of relativity of classical mechanics."

Daily life, too, has long ago learned to put up with the validity of this principle of classical mechanics without, we must admit, having become conscious of the real state of things. As you remember, we spoke some time ago of the fact that on a truck which travels along in a straight line with uniform velocity we can play at catching a ball in the same way as when we are quietly standing in a street. In both cases, moreover, a stone which we let drop falls perpendicularly to the ground, and our watch—a regular marvel of mechanical processes—works normally in either case. The falling stone, in particular, shows us the existing conditions very clearly.

We, who are travelling, naturally refer its motion to our carriage, *i.e.* in mathematical-physical terminology to a system of co-ordinates rigidly attached to this carriage. For what else does our statement, the stone falls in a straight line downward, mean, than that the line of its motion is perpendicular to the floor of our vehicle? An observer, however, who is standing in the street, and whom we pass in our journey, takes quite a different view of the matter. He sees the path followed by the stone as a peculiarly curved line which the mathematician calls a parabola. For him, whose system of co-ordinates is attached to the street, the stone, as you know, executes two motions simultaneously, first of all the falling motion perpendicularly downward, and secondly, together with the carriage, a uniform forward motion along the street. Both motions, in accordance with an important principle of mechanics, combine and appear to him a single motion, and the path of a combined motion of this kind has, as is shown by calculation and confirmed by observation, the form of a parabola. Therefore, while we who are travelling assert that the stone is descending in a straight line, the observer in the street asserts that it is descending in a parabolic path. The question is bound to arise which of the two parties is right. But who is to be the arbitrator? Both assertions are in complete harmony with the fundamental laws of mechanics. For us on the carriage, the stone was at rest as long as we were holding it in our hand. When we let it go the attractive force of the earth could act on it unhindered, and the falling motion caused thereby accurately followed the fundamental law applying to the effect of a force. For the observer in the street the stone held by us executed—just like ourselves and the carriage—a uniform rectilinear motion. When we released it the law of inertia tried to preserve this motion; but the law of force compelled the stone to descend to the ground. For his eye, these two motions constituted a single one,

just as is to be expected in mechanics, and, as a result, there appeared a parabolic line. By thus confronting the facts we see with striking clearness that it would be perfectly meaningless to ask about the "real path" of the falling stone. This "path" is an entirely relative conception. Relatively to the travelling carriage the path of the falling stone is a straight line, relatively to the street a parabola. Both statements, considered by themselves, are absolutely justified, and we are not entitled to speak of a contradiction between them.

In conclusion, let us once more briefly summarise our results. We started off from the idea that space, being absolutely at rest, supplies the immensely vast receptacle in which all natural phenomena take place. Step by step we then fought our way through to the result that the idea of a "space absolutely at rest" has to be dropped as meaningless with regard to all phenomena of motion, as long as they proceed on a rectilinear course with a velocity remaining permanently the same. Or, in other words, with regard to systems in uniform rectilinear motion, the conception of space appears to have become "relative."

In spite of this we need not, for the time being, give up the idea of absolute space. For, hitherto, we have expressly only spoken of uniform rectilinear motions; we never spoke of accelerated or retarded motions, nor of motions taking place in a curved trajectory. We know, however, from our own experience that, for instance, the catching of a ball on a roundabout in motion is not an easy matter. And we have been able to detect the motions of earth and sun just because the laws of mechanics lose their validity with regard to systems of co-ordinates moving in one of the ways just mentioned. Accordingly, the motion of such systems can quite well be ascertained. With regard to them the conception of a space at rest has a clear and definite

meaning, and the principle of relativity of classical mechanics must not be applied to them.

And, finally, there remains another good reason which justifies our hope of being able to preserve for space at rest its absolute character. Our conception of the world was only prepared to admit mechanical processes. How would it be if everything, after all, were not to be explained mechanically? If there were phenomena of a different, non-mechanical character, wouldn't they possibly enable us straightway to prove the existence of actual motions with reference to absolute space, and, thereby, the very existence of this " space absolutely at rest "?

VI

THE WORLD-ÆTHER

FROM your schooldays you remember perhaps a certain impressive experiment. In a glass vessel, closed to the air on all sides, was an electric bell, compelled to ring continuously by an electric current, the source of which was also fixed inside the glass vessel. Then, by removing the air from the vessel by means of an air-pump, you became distinctly aware of the fact that the sound of the bell was growing fainter and fainter, until it finally completely died away. From these observations you were taught to conclude that sound consists in a succession of condensations and rarefications of the air, and that, consequently, a sound wave cannot come into being in a space devoid of air (a so-called vacuum).

From the same experiment you may draw a further conclusion of equally high interest. As you know, the visibility of a body depends on the fact that rays of light proceeding from it are reaching our eye. In a completely dark room we are unable to see anything. Only when we admit light, and its rays are reflected by the objects, do these objects become visible to us. Now, in that space with its rarefied air you certainly could no longer hear the ringing of the bell; but you saw all the time how the hammer was knocking against the bell. Consequently rays of light must be capable of being propagated without air, they must be able to traverse a vacuum.

That this is actually the case is sufficiently shown by everyday observations, as you well know. For when you see the sun and the stars in the sky, this, after all, means nothing other than that their light is able to rush through the empty space of the universe and to cover the unthinkably great distances separating our earth from them. But how are we to conceive of this?

A simple solution would be the following: light consists of minutest material particles, let us say of tiny ball-shaped corpuscles which are continually being discharged by the source of light. These little balls obviously would be able to move through empty space, and if they were flung forth by the stars with sufficient impetus they might very well be able to reach the earth. Unfortunately this idea is untenable for other reasons. For from it would immediately result the fact that, by the addition of a second ray of light to one already in existence, an increase of light would be produced in all circumstances. For in that case the number of light corpuscles would be doubled, producing an effect on our eye twice as strong as before. But the experience of our physicists, based on experiments, has shown that, in certain very definite cases, by the combination of two rays of light into one darkness, *i.e.* absence of light may quite well be produced. As a result the theory of light suggested just now collapses as a matter of course, and we have to face the old difficulty once more.

Now imagine a motionless sheet of water, such as the surface of a pond presents on entirely calm days. By throwing a little stone into it you can give it wave-like motion, as you well know. In that case not the whole mass of water advances from place to place, as it might appear at first sight, but each individual particle of water remains in its place and executes a regular up-and-down motion perpendicularly to the original surface. You can illustrate this very easily by placing a little piece of cork on the water; it begins at once to dance

up and down quite charmingly without, at the same time, shifting to other places of the surface. The impression of an advancing motion of the waves is merely brought about by the fact that all the particles of water do not commence their dance at the same time. On the contrary, each individual particle follows the example set by its neighbour, only a little while later, and is, therefore, always delayed a little in its motion. The sum total of the individual motions following each other in this way then produces the impression of an advancing wave.

A short time after having thrown the first stone into the water, producing thereby the wave-like motion at the water's surface, let us drop, at the same place, a second stone into the water. Thereby we shall produce a new wave-like motion, and we will examine now how the two waves behave towards each other. In doing so we will keep our eye on the two principal cases. Let a certain particle of water owing to the first wave motion be just on the point of rising perpendicularly upwards from its position of rest on the surface. At this very moment the second wave arrives, in such a way that it, too, would like to force the particle of water in an upward direction. The particle of water would thus be driven upwards with double force, and would, consequently, rise twice as high. And since something quite analogous happens to all the other particles of water, owing to the fact that the two waves are placed one on top of the other, a single wave will spring from the surface exactly twice as big as each of the original waves. But the position will be quite different in the following case. Let once more, owing to the first wave, a certain particle of water be just on the point of rising upward. At this moment let the second wave arrive, this time with a tendency of pulling our particle of water in a downward direction. In this case, therefore, two equal forces will be acting upon the particle of water simultaneously

in exactly opposite directions. What is going to happen ?

Call to your mind an ancient and jolly game in gymnastics—the tug of war. There, at either end of a sufficiently long and strong rope, stand the players, and the party which displays the greater force will carry the other in the direction of its pull. But if both groups of players are equally strong, both remain standing in their places, however much they may pull and exert themselves.

Something entirely analogous is going to happen to our particle of water which is being seized by the two wave forces in the manner described. It will follow neither, but remain calmly lying at the surface. And all the other particles of water will do the same. Thus you notice that, in these circumstances, a double wave motion will lead to perfect calm on the water's surface.

This strange result gains increased importance, when you remember the fact mentioned above, that in certain cases, by the combination of two rays of light, darkness may result. For by such so-called experiments in interference, the possibility presents itself of conceiving light as a wave-like motion. Just because in wave-like motions two wave trains may easily produce a state of rest in the oscillating matter. In this case, therefore, the only question which remains open is as to what would have to be regarded as the carrier of the vibrations of light. For without some kind of oscillating matter any wave structure would appear impossible, as a matter of course. The air need not be considered for this purpose, as you know from the experiment at school with the bell in a glass vessel from which the air had been removed.

Thus physics found itself faced by the task of having to discover some substance which would fill the whole universe without leaving a gap. For the rays of light travel through the whole universe, and they are capable of penetrating through matter, such as glass and water,

for example. Consequently the substance looked for would have to be capable of penetrating through glass and water too—indeed through any kind of matter. It must not be removable by the best air-pumps ; it must be without weight, because otherwise we should know of its presence by other phenomena. And, above all, it would have to possess the peculiar quality, in spite of its omnipresence, of not disturbing in the least the motions of the celestial bodies. For if, for instance, the earth on its way through that substance should experience friction, her motion round the sun would become slower and slower in consequence of this resistance. In the course of time, therefore, a year would become longer and longer. Such a phenomenon, however, has never been observed by astronomers, either with regard to the earth or any other planet, although they employ the most sensitive methods, so that even the slightest influences would not escape them. Briefly speaking, that substance must combine the strangest qualities in itself. And yet it is indispensable to assume its existence, because without it the propagation of light would remain an unsolved riddle. So—with a heavy heart, let it be said—it has been received as an important constituent into the physical system, and has been called by the name of World-Æther.

This world-æther has become a real child of sorrow in theoretical physics. For its peculiarities are by no means exhausted by the qualities mentioned above. If it really is to act as the carrier of light waves, it has to be regarded as a solid body with regard to its elastic behaviour. For vibrations of the character represented by light vibrations are only possible inside a solid body, as has been shown by accurate investigation. The world-æther, a solid body, through which our earth is flying without being hindered by it in the slightest degree ! This sounds as incredible as could be. One has tried to find a way out by the assumption that the earth, as well

as all bodies moving in the æther, carry part of the æther along with them. It is a pity that this assumption has to be entirely rejected. For if it were correct to say that the earth is dragging along a part of the æther, a ray of light coming to us from the sun would be deflected from its course by a definite amount. But even the most delicate measurements have never established the faintest trace of such a deflection.

I shall be glad to spare you a detailed account as to how various physicists, in the course of time, have exerted themselves to reconcile the properties of the æther which contradict each other in so many ways. One may safely risk the statement that there are as many different theories of æther as there are eminent physicists. Simply because every one of them formed his own ideas about it, refusing at the same time to recognise those of others. Still, all physicists were agreed on this point, that in spite of all these difficulties the world-æther was something really existing.

The theory of æther indeed carried with it one immense advantage—it led to a wonderful unification of our conception of the world. Not only light, but also the radiation of heat and the electric vibrations, appeared to be changing wave-like states of the æther. Not only this, but the æther was able to accomplish much more. If you rub an ebonite stick with a woollen rag, and hold it afterwards close to small pieces of paper, these pieces will fly on to the stick, and they will do so even if the air should have been removed between stick and paper. This phenomenon is well known to you as an elementary experiment in electricity. But what is it that induces the pieces of paper thus suddenly to fly towards the stick? Because it has become electrified, you suggest. That is indeed so, but how do the pieces of paper get to know about the electric charge of the ebonite stick? In other words, how can the electric force act in the distance through empty space? We

shall have to defer the answer for the moment. Only the world-æther will help us out of the difficulty. For if we assume that this æther, by the electric charge of the ebonite stick, is being thrown into a particular state of tension, we immediately see that, by an extension of this state of tension through the whole neighbourhood of the ebonite stick, the pieces of paper may be seized by it and attracted towards the stick. Therefore, in order to interpret the phenomena of electricity, we need not make use of the inconceivable idea of a force acting at a distance. Æther acting as the carrier of electric action immediately makes these phenomena conceivable. The same applies to magnetic phenomena, and summarising the facts we may be allowed to say, by the assumption of the world-æther, we render possible the understanding of optical, electrical, and magnetic processes. We have to admit, however, that we pay for this understanding by concessions as to the nature of æther which we should never be induced to grant to other substances. World-æther thus becomes a substance of a very special kind, a substance omnipresent, all penetrating, of extremely fine structure, through which the heavenly bodies are moving like grains of sand through a sieve.

VII

THE EXPERIMENT OF MICHELSON AND MORLEY AND ITS INTERPRETATION BY H. A. LORENTZ

THE principle of relativity in classical mechanics had severely shaken our old and familiar conceptions of space. For, according to it, it was to be impossible to distinguish between a space at rest and a space in uniform rectilinear motion! No doubt, only with regard to purely mechanical processes, a relativisation of our conception of space had to be considered. But have we not frequently pointed out already that, at bottom, all phenomena of nature admit of a mechanical interpretation? And if this be so, are we not then clearly compelled to banish absolute space definitely from our conception of the world?

By the assumption of a world-æther theoretical physics had been enabled to explain optical, electrical, and mechanical phenomena by mechanical processes. Thus it became possible to divide the whole of physics into two large provinces, firstly, the mechanics of material bodies, and secondly, the mechanics of the world-æther. In the mechanics of material bodies the principle of relativity held; was it valid, too, with regard to the mechanics of the æther?

At first this question had obviously to be answered in the negative. For the world-æther was to be an omnipresent substance, absolutely at rest, filling the

whole space of the universe without a gap, a substance through which all bodies are moving without carrying it with them to the slightest extent. Therefore it had a well-defined meaning to call the world-æther the space absolutely at rest, and a system of co-ordinates whose origin was fixed in the world-æther would distinguish itself undoubtedly from the systems of co-ordinates attached to any celestial bodies. In other words, it ought to be possible to establish, somehow or other, the motion of, let us say, the earth relatively to the world-æther. In w h a t way? That is what we will now try to explain.

Light, considered as a change of state extending in wave-like manner, is propagated in the æther with equal velocity in all directions. The velocity reaches the inconceivable rate of 300,000 kilometres per second. Since the equator of the earth is roughly 40,000 kilometres long, a ray of light could travel round the earth almost seven and a half times in a single second! It is self-evident that the value of the velocity of light only holds with reference to a system of co-ordinates which is at rest in the world-æther. A physicist who undertakes to measure it would have to proceed somewhat in the following manner: at a certain moment he allows a ray of light to start in any direction he may choose, from a point at rest in the æther, fixed in his system of co-ordinates by three definite co-ordinates. He then measures the co-ordinates of the point which the ray of light has reached after the lapse of exactly one second. From the results obtained for the co-ordinates, the distance between the beginning and the end point of the light path can be calculated, and this distance represents the path completed by the ray of light in a second, *i.e.* the velocity of light.

Now think of the earth moving through the world-æther. In its orbit round the sun it advances by about thirty kilometres per second. What opinion will a

physicist form about the velocity of light as seen from the earth?

First of all, this much is absolutely certain: from the moment a light ray has been produced on the earth, no matter how, it belongs to the world-æther. For, according to general opinion, it represents nothing but a quite definite change of state of this æther! Whatever this change of state may be like in detail does not matter in the least, we are satisfied with the fact that the velocity of its transmission through the æther amounts to 300,000 kilometres per second. One second after its birth the ray of light has, therefore, reached a point of the world-æther 300,000 kilometres distant from its birthplace, *i.e* from that place of the æther where the source of light happened to be at the moment when the ray of light came into being. Now, during this one second, the source of light rigidly connected with the earth has, with regard to the æther, advanced by thirty kilometres. So if we send forth a ray of light exactly in the direction in which the earth is moving, at the end of the first second the distance between the source of light and the foremost point of the ray of light is only 300,000−30, *i.e.* 299,970 kilometres. A physicist, therefore, who happens to be on the earth, and consequently refers his measurements to a system of co-ordinates rigidly attached to the earth, is, for this reason, bound to arrive at the result that the velocity of light in the direction of the motion of the earth amounts to 299,970 kilometres per second. But if, in a second experiment, he sends a ray of light in a direction opposite to the motion of the earth, at the end of the first second the distance between the source of light and the foremost point of the ray of light does not only amount to the 300,000 kilometres which the ray of light has travelled in the meantime, but, in addition, to another thirty kilometres which the source of light has travelled during that one second in an exactly opposite direction. The

physicist, therefore, will determine the distance travelled by light to be 300,000 + 30, *i.e.* 300,030 kilometres per second. In other words, on the earth the velocity of light in the direction of the motion of the earth must be thirty kilometres less, in the direction opposite to the motion of the earth thirty kilometres more than the normal velocity of light in the æther.

Perhaps our last considerations have made your head swim a little. But just think of the simplest observation which you can make yourself at any time during a railway journey. If, from the window of your compartment, you try to judge about the velocity of passing trains, you will arrive at results essentially different from those arrived at by a man standing on the line. Any train travelling in the same direction as you do will appear to you to be moving much slower than it does to the man on the line, slower exactly by the velocity at which your own train is travelling. How this happens you will probably understand from the following description. Let the train, owing to its velocity, travel twenty carriage windows past an observer at rest during one second. No doubt the train would travel these twenty carriage windows past you, too, in the course of a second. But your train prevents it from doing so. For—owing to *its own* velocity—it carries you along in the same direction for a distance equal to twelve carriage windows, so that the other train only travels eight windows past you. If the other train advances to meet yours from the opposite direction, it presents to you twenty of its carriage windows per second, at the same time *your* train takes you past another twelve of its carriage windows per second, so that, in the course of a second, you see altogether thirty-two carriage windows—whereas the man on the line sees as before only twenty carriage windows per second of the second train.

Thus we arrive at the important result that on this earth the velocity of light must depend upon the direction

in which the propagation of the rays of light takes place. Of course even the greatest difference that can be observed in this respect will not be very appreciable, considering the high figures we have to deal with here. All the same, modern experimental physics possesses methods sufficiently sensitive to enable us to test our conclusions by experience. And the whole apparatus needed for this purpose can easily be accommodated in a large room. This circumstance compels us to insert here a remark of some importance. As you know, the yearly orbit of the earth round the sun has the shape of an ellipse. The motion of the earth, therefore, actually follows a curved line; moreover, it takes place with non-uniform velocity according to the second of Kepler's laws mentioned previously. In spite of this, for the experiments planned we have to accept it as a uniform rectilinear motion. For a mathematician can strictly prove to us that those small portions of the earth's orbit, which we shall have to consider in this connection, are straight lines, and are travelled by the earth with absolutely uniform velocity.

Now in order to be able easily to find out the difference of velocity expected by them, the American physicists, Michelson and Morley, arranged an experiment, the essential features of which are shown in Figure 4. The rays of light coming from their source in L strike the transparent mirror S, which splits them up into two beams. One of these moves in the direction of the motion of the earth (indicated by an arrow) to the mirror Q, where it is reflected, and returns to S in a direction opposite to the motion of the earth. A second beam of light starts from S perpendicularly to the direction in which the earth is moving, arrives at the mirror P, which is exactly as far from S as Q is, and is then reflected towards S. Thus both beams of light meet again in S, and the question arises as to what is going to happen there.

Let us, first of all, consider the first beam of light, moving from S to Q. As a result of the motion of the earth taking place in the same direction, its velocity will be reduced, but it will be increased on its way back from Q to S. Now, in all probability you will feel inclined to

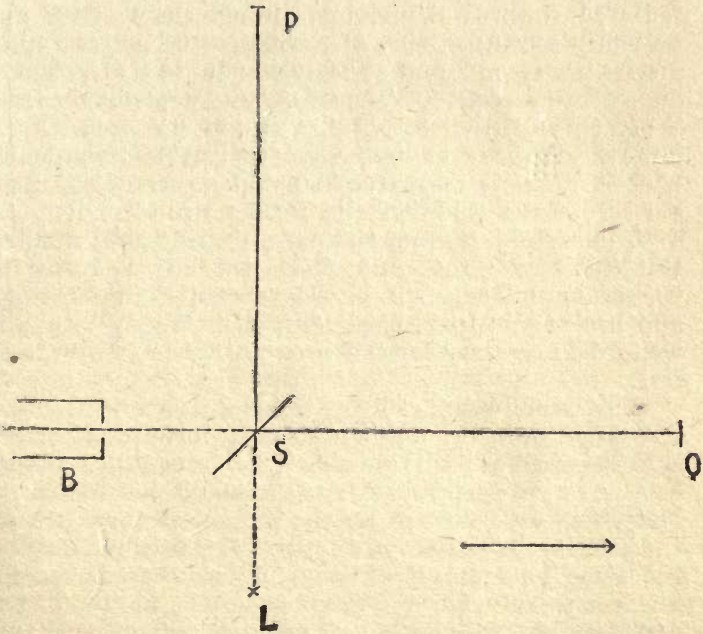

FIG. 4.

think that the reduction of the velocity on the way forward will be exactly balanced by the increase on the way back. But just consider the point by the help of a very illuminating arithmetical example. Let us assume the velocity of the earth to be, not thirty, but 100,000 kilometres per second. Furthermore, let the

distance QS not be eleven metres—such as it was when the experiment was actually performed—but 300,000 kilometres. Since under these circumstances the velocity of light in the direction from S to Q would only amount to 200,000 kilometres, the light ought to arrive in Q one and a half seconds after having left S. Now it is reflected towards S, moving, in agreement with our previous considerations, at a velocity of 400,000 kilometres per second, and so arrives again in S after three-quarters of a second. The whole way accordingly from S *viâ* Q, and back to S, takes $1\frac{1}{2}+\frac{3}{4}=2\frac{1}{4}$ seconds. If, however, the ray of light were to travel forward and backward at its normal velocity of 300,000 kilometres per second it would arrive in S after two seconds.

The real figures are, of course, considerably smaller. But this leaves the result itself untouched; owing to the motion of the earth, in order to travel from S to Q, and back again to S, light takes a little longer than it would be necessary for this process if the earth were at rest.

The second beam of light, which, up to now, we have not taken into account, on its way forward, *i.e.* from S to P as well as on its way back, *i.e.* from P to S, moves in a direction perpendicular to the motion of the earth. Here, too, an influence by the motion of the earth on the velocity of the light will occur. But it will be exactly the same both ways, although different from the influences produced in the direction of the motion of the earth, so that, for simplicity's sake, we may disregard its existence altogether. So we shall quietly assume that this second beam of light completes its course with the normal velocity of light. Hence it will arrive in S a certain time sooner than the first beam, and, as a result of this, very definite phenomena of interference will appear, which can be calculated theoretically, and can be observed and measured at point B by means of special contrivances which we need not discuss in detail.

So much for theoretical considerations resulting from the assumption of a world-æther as the carrier of optical phenomena, and from the actual motion of the earth round the sun. The practical experiment, however, performed by Michelson and Morley, and frequently repeated since, does not show the slightest trace of the expected effect. Yet the method adopted for the experiment was of such a sensitive character that even the hundredth part of that effect would necessarily have been detected without fail.

Now, as you know, a principal part had been assigned to the world-æther, not only in connection with optical, but, above all, with electrical phenomena. So the possibility presented itself of devising processes in the realm of electricity, the course of which must be influenced by the motion of the earth. Here we need not go more closely into the details of these electrical experiments. They all, without exception, led to the extremely strange result that the influence expected from the motion of the earth made itself in no wise felt. But how was this to be explained?

A first answer to this question was given by the Dutch physicist, H. A. Lorentz. Lorentz adheres strictly to the æther theory. But, in order to explain the unexpected result of the experiments just mentioned, he introduces a hypothesis which looks extremely strange. He assumes that every body, however solid and rigid it may be when it is moved through the world-æther in the direction of its motion, suffers a contraction, whilst it retains its original form unchanged in the direction perpendicular to its motion. Thus the earth, too, and on the earth, the distance between the two mirrors, S and Q, used in Michelson-Morley's experiment, are said to contract in the direction of their motion. And this contraction is said to be exactly big enough to counterbalance the difference of time which we had expected to exist between the path travelled by the

light from S to Q, and back, and the path travelled by the light from S to P, and back. On this assumption it becomes immediately evident that both beams of light will meet again in S at the same moment. For even if the changes of velocity continue along the line SQ, the contraction of this distance brings it about that the light travelling forward and backward between S and Q takes exactly the same time as travelling forward and backward the distance SP which remained unchanged by the motion of the earth. Consequently it is impossible to prove, by Michelson-Morley's experiment, the actually existing influence of the motion of the earth on the velocity with which light is propagated. By analogous assumptions Lorentz succeeded, further, in bringing into harmony with theory the electrical experiments mentioned previously.

Of course you at once raise the question whether it should not be possible, by direct measurements, to show this contraction of the distance SQ between the mirrors. But this is utterly impossible. For in order to measure the length of SQ you would have to place a measuring-rod alongside SQ to see how many times it could be marked off on this distance. But as soon as you turn the measuring-rod in the direction of SQ it, too, suffers a corresponding contraction as it moves through the world-æther in the direction of its own length. As a result, you would obtain the same length, although, in reality, a contraction has taken place.

So you easily see that a confirmation, or a refutation, of Lorentz's hypothesis by experiment is, unfortunately, impossible. And yet you will hardly be satisfied by what it states. Men of science feel similarly towards it. The mysterious world-æther, with this influence on the size of bodies moving through it, had given them a new riddle to solve. And thus, to use the words of the mathematician, Hermann Weyl of Zürich, the problem

arose "for the mechanics" of the æther to explain this remarkable effect on matter, too, which occurs in such a way as though the æther, once and for all, had made up its mind: "You blessed physicists, you are not going to catch me."

VIII

EINSTEIN'S SPECIAL PRINCIPLE OF RELATIVITY

THE result obtained by the Michelson-Morley experiment admits, however, of an interpretation essentially different from the one given by H. A. Lorentz.

Call back to your mind once more the contents of the principle of relativity in classical mechanics. All purely mechanical processes are absolutely independent in their course of the system of co-ordinates to which they are referred as long as one limits oneself to systems at rest, or to systems having uniform rectilinear motion. Hence, by purely mechanical experiments we can never arrive at a decision as to whether we are in a system of co-ordinates at rest, or in one moving uniformly in a straight line. This is so because in systems moving in this way mechanical phenomena take place exactly in the same manner as in space at rest.

The experiment performed by Michelson-Morley had shown something absolutely similar with regard to the propagation of light. For if light on the earth, too, is propagated with the same velocity in all directions, the earth, in this respect, is not distinguished in the slightest degree from a celestial body embedded motionless in the world-æther. So it becomes impossible to prove the motion of the earth by an investigation of phenomena connected with the propagation of light. The same applies with regard to the electrical experiments which

were made in pursuance of the Michelson-Morley experiment; they, too, did not admit of any interpretation confirming the fact that the earth, during short periods, is in uniform rectilinear motion with regard to the æther.

So it does not seem to be far-fetched to generalise the principle of relativity of classical mechanics, expressing it, in a preliminary way, in the following terms: all natural phenomena—not only the purely mechanical ones, but the electrical, magnetic, and optical as well—take place in the same way, whether referred to a system of co-ordinates at rest, or one in uniform rectilinear motion with regard to the system at rest. This is the conclusion drawn by Albert Einstein in 1905 from the result of the Michelson-Morley experiment. It is called, to-day, the special principle of relativity.

The formulation of the special principle of relativity, therefore, is based on experimental facts of experience. By this circumstance it differs essentially from the principle of relativity in classical mechanics. For the latter, as you will remember, had been found by merely theoretical calculations. When discussing this matter we had tried to make clear to ourselves the fact that the fundamental laws of mechanics must take the same form for systems of co-ordinates at rest, and for those in uniform rectilinear motion. From this we had drawn the further conclusion that all mechanical processes will take place in a corresponding manner in both kinds of systems. The accuracy of this conclusion we, afterwards, saw confirmed by experience. But now, Einstein's principle of relativity is urged upon us by experience, since Michelson-Morley's experiment with regard to the phenomena connected with the propagation of light reveals the equivalence of a system of co-ordinates in uniform rectilinear motion, and a system at rest. The principle strictly contradicts our previous theoretical

ideas since, as you remember, the actual result of Michelson-Morley's experiment appeared absolutely mysterious to us. Therefore we shall have to try to get our original theoretical considerations, by altering them somehow or other, into harmony with the statement contained in the principle of relativity. That the solution of this problem is possible was shown by Albert Einstein, too, and it is just this fact which constitutes the gigantic importance of the service he has rendered to science.

The basis of our conception of the world, up to now, had been a space, absolutely at rest and filled with æther. A system of co-ordinates, with its origin resting in this space, was said to be distinguished from all systems of co-ordinates in motion by the fact that the laws of nature are fulfilled in it with absolute accuracy. But, as we know by now, an infinite number of systems of co-ordinates will satisfy this condition. Hence, it would be utterly meaningless arbitrarily to single out one of them, and then call it the system absolutely at rest. However hard it may appear to us—there is no other way out of the difficulty than to give up for good our old and familiar conception of space. We have no longer a right to speak simply of space at rest and space in motion, but only of *spaces moving relatively to each other*. The motion of a space in itself is a meaningless notion which we cannot define. Only with reference to another space is it possible to speak of a particular space as being in motion, but in this case we are equally at liberty of conceiving the latter as being at rest, and the former as being in motion relatively to it. Once we see this clearly we are, of course, faced by the necessity of giving up our provisional wording of Einstein's principle of relativity, and must formulate it in the following way: *The laws of nature governing the course of natural phenomena are absolutely independent of the fact as to whether*

they are referred to one or the other of two systems of co-ordinates in uniform rectilinear motion relatively to each other.

Our next task will be to modify our original conceptions regarding the propagation of light, and to bring them into harmony with the demands of the principle of relativity. We shall have to make clear to ourselves which suppositions will have to be fulfilled so that the propagation of light can take place with perfect uniformity with reference to two systems of co-ordinates in uniform rectilinear motion relatively to each other, in such a way that not a single direction is distinguished from other directions by a special value for the velocity of light, and that, thereby, an agreement with the Michelson-Morley experiment is established.

Apart from this, several facts of physical experience compel us to add another fundamental postulate, namely, the postulate that light in empty space is always propagated with the same velocity, no matter whether the body emitting the light is at rest in space, or is moving in any way whatever. Let us consider, therefore, two systems of co-ordinates which we will briefly call system O and system O_1, moving relatively to each other in a straight line with an unchanging velocity of 100,000 kilometres per second. Our contention, then, is that any ray of light whatsoever will be propagated with the same velocity with reference to both systems, independently of the fact on which of the two systems the source of light may be.

About the character of the process by which light is propagated we will not suppose anything in particular. Since it has not been possible by any means whatsoever to prove the existence of a world-æther, formerly assumed for this purpose, and, since the properties ascribed to it

have always appeared extremely strange to us, there is not the slightest reason why we should retain it any longer. All we know is that light is propagated in empty space, and the belief in the existence of a world-æther becomes unnecessary.

But now let us look at the consequences which result from the validity of Einstein's principle of relativity as well as from the assumption of a constant velocity of light in empty space. If we are to determine the velocity of light in the above-mentioned system of co-ordinates O, nothing else is meant than that we are to establish the distance travelled by light in the course of a second in the system O. So we have, first of all, to choose two points, A and B, which have a fixed position with reference to the system O, the distance between them being known to us. We then have to find out the time light takes in travelling from A to B. The easiest way to do this would be to send, at a definite moment, a ray of light from A, and then to look at what time this ray of light arrives in B. But for this purpose we need two clocks going exactly at the same rate, and with their pointers set exactly alike. Let us assume as given that they are going at the same rate. But how are we to arrive at the same position of the pointers if one of the clocks is at A, the other at B? No doubt you imagine the thing to be very simple. You would just carry the clock from B to A, set it to the same time as that shown by the clock there, and then carry it back to B. Of course, it is quite feasible to get the same position of the pointers for the two clocks at A, because the idea of the simultaneousness of two events in one place has a definite meaning to us. But when you propose to carry back the second clock, after having regulated it, to the point B, you tacitly assume that the position of the pointers is not interfered with by the clock's motion from one place to the other. As long as we know nothing definite about this fact we had

better avoid this assumption, and be on the look-out for a possibility of obtaining the same position of the pointers even if both clocks remain in their places. As a matter of fact, such a possibility presents itself because the principle of relativity postulates that light has to be propagated with the same velocity in all directions with reference to the system O. First of all, therefore, we measure the distance between the two points A and B by marking off with a standard measuring-rod—let us say a rod one kilometre long—the line AB, beginning at A until we reach the point B. The number of times we have placed the rod along the line will give us the measure of the distance ; let it amount to exactly 300,000 kilometres. We then determine the mid-point M, in the line AB, which can easily be done with the greatest of accuracy by a simple geometrical construction. At this point M we now place two small mirrors, as shown in our diagram 5, so that the two points A and B are visible in them simultaneously. We further ask an assistant at A to send a ray of light in the direction of B at the moment when the clock at A points exactly at twelve. In the same way a second assistant at B is to send a ray of light in the direction of A at the moment when the clock at B points exactly at twelve. Now, since M, where we are situated, is at a distance of exactly 150,000 kilometres from both A and B, and since light, in accordance with the postulate of the principle of relativity, is propagated with equal velocity from A to M, and from B to M, we are bound to see the two light signals arrive simultaneously, provided the pointers of the clocks at A and B have got the same position. If this should not be the case we can ask our assistant at B to change the position of the pointers on his clock until the simultaneous arrival of the rays of light at M is secured. In this way we shall be certain that the pointers of the two clocks at A and B actually possess the same position.

Aiter having completed the preparations described so far, the velocity itself can be measured without much trouble. We only have to give the following instructions to our two assistants: punctually at twelve o'clock a ray of light is to be sent from A to B. At B the time when the ray of light arrives there is to be ascertained. In the instance chosen by us the ray of light will arrive at B exactly one second past twelve o'clock, and thus we

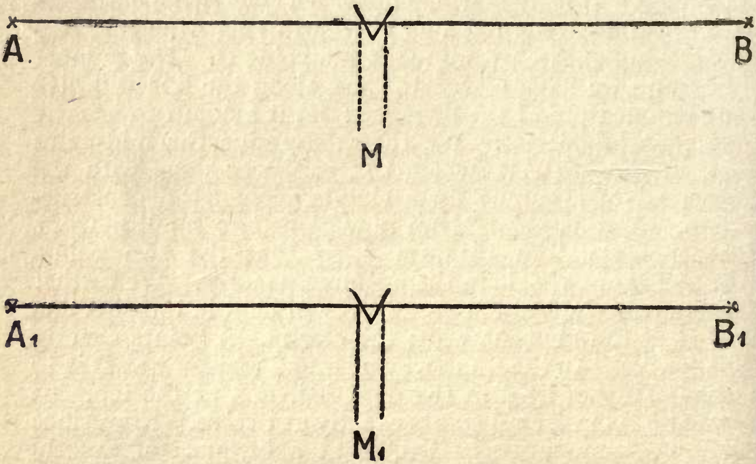

FIG. 5.

obtain, for the velocity of light, the value of 300,000 kilometres per second.

We shall have to proceed exactly in the same manner when we happen to be in the second system marked O_1, and desire to measure the velocity of light there. We shall have to choose two points, A_1 and B_1, at rest with reference to the system O_1, then measure, in kilometres, the length of the connecting line A_1B_1 by repeatedly placing our standard measuring-rod along it, and, finally, find point M_1, half-way between A_1 and B_1. We then place

a clock at each of these two points, both clocks being exactly alike, and their pointer position regulated by two assistants in the manner described just now. This method will be perfectly reliable again, because, *in accordance with the postulate of the principle of relativity*, with reference to the system O_1, too, light is propagated with the same velocity in all directions. As soon as the clocks are regulated we are able, just as before, to determine the velocity of light, and we will assume that here, too, we find, as a result, a velocity of 300,000 kilometres per second.

So far, then, everything would be in perfect order. But now we two are going to separate. You may remain in the system O together with two assistants placed at A and B whilst you proceed to the mirrors fixed at point M. I go across to the system O_1, and take two assistants with me also to look after the source of light and watch the two clocks placed in A_1 and B_1. Now I am going to watch you at your work. Do not forget that you are in the system O, whereas I am in the system O_1. *For you* the system O is at rest, while the system O_1 is moving; *for me*, on the other hand, the system O_1 is at rest, while the system O is moving. Both ways of looking at the situation are equally justified on account of the relativity of the conception of motion. After you have had your clocks regulated you would like to test them once more as to their accuracy. To do this you give your assistants the necessary orders and ascertain to your satisfaction the simultaneous appearance at M of the light signals emitted from A and B, which means that the clocks placed at those points show exactly the same positions of their pointers. But if you were to think that I should arrive at the same result you would be in grave error. *For me*, as you are aware, your system O is in uniform rectilinear motion as compared with my system O_1, and we will assume that the direction of this motion coincides with the direction from B_1 to A_1

(*upper* arrow in Fig. 5). Consequently, for me, your mirrors at M are moving towards the ray of light coming from A. Now, since with reference to my system O_1 the velocity of the ray of light is to be the same as with reference to your system O, it is bound, *for me*, to arrive at M sooner than the ray of light emitted from B from which—*as judged by me*—the mirrors are moving away. Of course the result will be that in *your* mirrors at M I shall see first the appearance of the ray of light arriving from A, and considerably later that of the ray of light coming from B. Thus I shall be compelled to contend that your two clocks cannot possibly have the same position of their pointers. But you, too, will make exactly the same statement with regard to *my* clocks after I have regulated them *for my system*. Because, *for you*, *my* mirrors at M_1 are moving in the direction from A to B (*lower* arrow in Fig. 5). Consequently the ray of light emitted from the point B_1 in *my* system O_1 will, *for you*, reach the mirrors at M_1 considerably sooner than the ray of light emitted from the point A_1. So if I, relying on my observations, maintain that my clocks at A_1 and B_1 are alike in the position of their pointers, you, relying on your observations, will maintain that they differ as to the position of their pointers. Or, in other words, the same two events—namely, the appearance of a light signal at the two points A_1 and B_1 of *my* system O_1—which I call simultaneous, you will call non-simultaneous on the strength of your observations. And, vice versa, I become aware at different times of two events—the appearance of a light signal at the points A and B of *your* system O—the simultaneousness of which you maintain.

This is the first important result to which our investigation has led us. We were out to discover what consequences may be derived from the validity of Einstein's principle of relativity, and from the validity of

the principle of the constant velocity of light in empty space. First of all, we have found that, in this case, we shall have to give up the idea of "simultaneousness in itself," because it becomes meaningless. Thus the idea of simultaneousness is a relative conception. Two definite events which, as seen from a definite system of co-ordinates, happen at the same moment, may no longer be called simultaneous when they are seen from another system which is in motion with reference to the former system.

After we have become absolutely clear about this point you imagine yourself once more to be in the system O, whilst I remain in the system O_1. If you have a ray of light sent from A to B then, in accordance with the demand of the principle of relativity, it takes the same time in travelling this distance as when you are sending it from B to A, *i.e.* exactly one second in both cases. But this is true only for you, and such observers as may be with you in the system O. I, on the other hand, take an essentially different view of the matter, on observing it from my system O_1. For if, for instance, at the moment when A and A_1 are opposite each other, our corresponding clocks are pointing at twelve, and you, at this moment, send a ray of light in the direction B, this ray of light will arrive opposite my point B_1 when my clock, placed there, points to one second past twelve. But your point B is travelling towards that ray of light, since your whole system O, relatively to my system O_1, is moving in the direction from B to A. Consequently my clock at B_1, on the arrival of the ray of light at your point B, will show less than a second past twelve, so that, for me, the light takes less than a second to travel the distance from A to B, *i.e.* in your system. Now, if a mirror is fixed at B, and reflects the ray of light, immediately on its arrival, back towards A, your point A will be travelling away from the ray of light. Hence, the ray of light, as judged

from my point of view, will take more than a second to get from B to A.

Thus, as a second result, we see that not only the idea of simultaneousness, but also the idea of duration, will have to be relativised. For if two processes—in this case the motion of light from A to B on the one hand, and from B to A on the other—are of the same duration with reference to a particular system of co-ordinates, but seen from another system which is moving with regard to the former, are judged as being of different duration, then the conception of a definite "duration in itself" loses all sense and meaning. Therefore we are constrained in connection with all statements of time to mention, in future, the system of co-ordinates with reference to which they are measured. Only by knowing the system of co-ordinates shall we be in a position accurately to value the statement made about the duration in question.

But we have still not yet come to the end of the deductions resulting from the validity of Einstein's principle of relativity. There remains a third one, the cogency of which, however, you will now easily see. As we mentioned previously, you can find the length of the distance AB, as you are at rest in your system O, by marking off a standard measuring-rod along it. In the instance chosen by us, it was to amount to 300,000 kilometres. But if I, from *my* system O_1, wish to measure *your* distance AB as well, the following method presents itself to me: at a particular time I find those two points of my system opposite to which, at that moment, your points A and B are exactly placed. Thereupon I measure, with my standard measuring-rod, the distance between the two points of my system obtained in this manner, and the result of this measurement must represent the length of the distance AB I was looking for. At first you are likely to think that, in this way, both of us will arrive at equal values for the distance

AB. But consider the matter more carefully! I meant to find *simultaneously* the respective positions of your two points A and B with reference to my system O_1. But, as we know, it is a delicate matter to determine the simultaneousness of two events. For what I, from *my* system, call simultaneous, happens at different times for you. Thus it seems only too probable that our two measurements of one and the same distance AB, which is at rest in your system, will lead to absolutely different results. But the conditions actually prevailing can only be understood by treating the problem mathematically, and, as a matter of principle, you know that I do not wish to bother you with mathematics. As far as we are concerned the result alone matters, and it turns out in practice that the length of a certain distance is judged differently according to whether the measurement is carried out in a system of co-ordinates at rest with reference to the distance to be measured, or in a second system which is in motion with reference to the former system. Now let us take a comprehensive view of our reasonings up to this point. Stimulated by the result of the Michelson-Morley experiment, Albert Einstein, in 1905, laid down the following two principles:

1. The principle of relativity. The laws according to which physical phenomena run their course are independent as to which of two co-ordinate systems, moving uniformly and rectilinearly with respect to each other, the phenomena are referred.

2. The principle of the constancy of the velocity of light. Each ray of light travels, in empty space, with an absolutely definite velocity, no matter whether the ray of light be emitted from a body at rest, or from one in motion.

By an admission of these two principles we are led to the following deductions:

1. A statement asserting the simultaneousness of two

events can only be made with reference to a definite system of co-ordinates. With reference to another system of co-ordinates, in uniform rectilinear motion with respect to the former, the same events happen at different times.

2. The time-interval between two particular events is judged differently from two systems of co-ordinates moving relatively to each other.

3. The space-interval between two particular events is judged differently from two systems of co-ordinates moving relatively to each other.

Considering this state of things, you naturally at once raise the further question as to what will be the actual relations existing between the statements of time and distance with respect to two systems of co-ordinates moving with a definite velocity relatively to each other. These relations are to be found by mathematical methods. To understand this you only have to recall our previous considerations, in connection with the theory of the æther, which dealt with the propagation of light with reference to a system of co-ordinates rigidly attached to the earth. On that occasion we arrived at the result that, in the direction of the motion of the earth, light is propagated with a velocity equal to the normal velocity of propagation, reduced by the velocity of the earth; that, on the other hand, the normal velocity of light is increased by the velocity of the earth, as soon as we examine light moving in a direction opposed to the motion of the earth. But this way of looking upon things was only admissible because, unconsciously, we made two fundamental suppositions. First of all, we assumed that the measure for the lapse of time is exactly the same for space "at rest" as it is for the earth in motion; secondly, we took the space-interval between two points of a rigid body to be a strictly defined magnitude, which appears absolutely identical in empty space and on the moving earth. For we intended to measure, in

both cases, with the same measures! But now we have seen that these two assumptions are inadmissible, and that, on the contrary, the time-interval between two events as well as the space-interval between two points of a rigid body depend on the state of motion of the system of co-ordinates with reference to which these magnitudes are being examined. So, if we measure the velocity of light on the earth we shall, on account of the motion of the earth, as compared with space "at rest," find other times and other distances than if we were watching the same process from space "at rest." By claiming now that in both cases equal values are to result for the velocity of light in all directions we have an opportunity of finding, by calculation, the relations we are looking for between measurements of time and distance made in two systems of co-ordinates moving relatively to each other. These calculations were carried out by Albert Einstein. As a result he obtained certain mathematical formulæ. These formulæ enable us to transfer, by calculation, the measurements of time and distance found, let us say, in our system of co-ordinates O_1, to the system of co-ordinates O.

To give you some idea as to the results of Einstein's formulæ, let us now consider a few cases of special importance. You may again stay in the system O, while I am in the system O_1. If at any point of your system you place a clock which, for you, is ticking at the end of every second, then I, from my system, shall never take this clock for a seconds-clock. For since, together with you and your whole system, your clock is moving with respect to me, it will appear to me retarded according to Einstein's formulæ. Thus, if I compare your clock with a second clock, indicating seconds, which in my system O_1 is fixed at rest, my clock will tick for me in quicker succession than yours. Vice versa, when you compare our two clocks with

each other you will be of opinion that my clock goes more slowly than yours. Simply because, for you, my clock is moving, yours at rest, and a moving clock always goes more slowly than one at rest. You see, therefore, that our views are entirely reciprocative. Which of us is "really right" is an idle question. The duration in time of any process is merely a matter of view-point, and you have the same right to consider my clock the slower one, as I have to hold the opposite view. The two statements do not contradict each other any more than your assertion that your system O is at rest while my system O_1 is moving contradicts my assertion that my system O_1 is at rest while your system O is moving. For, according to Einstein's principle of relativity, not only the conceptions of rest and motion, but the conception of duration and time, also, are relative conceptions.

Applying the knowledge thus acquired, you will no doubt clearly see by now why, on a previous occasion, we did not—what at first might have seemed natural to do—set the pointers of our clocks in such a way that we, placing the clocks somewhere side by side, just brought their pointers into the same position, and then took each clock to its destination. For the motion of the clocks would have retarded their movements according to the velocity of the motion. Thus the positions of the pointers, originally the same, would have been more or less disturbed by the time of their arrival at their destination. In order to avoid this mistake we had to adopt the method of regulating the pointers by means of rays of light, which to you seemed rather a roundabout way.

For the purpose of measuring the velocity of light we had, on a former occasion, measured in our systems O and O_1 the distances AB and A_1B_1 respectively, both of them supposed to be lying exactly in the direction of the mutual motion of our systems. For you, your

distance AB was to be 300,000 kilometres long; for me, the length of my distance, A_1B_1, was to have the same value. Now since, for you, your distance AB is at rest, while my distance A_1B_1 is moving, you arrive—again according to Einstein's formulæ—at the conclusion that my distance A_1B_1 is shorter than your distance AB. Vice versa, your distance AB appears to me shorter by a definite amount as compared with my distance A_1B_1, since, for me A_1B_1 is at rest and AB is moving. Again, you must not ask about the real length of the distances AB and A_1B_1, because the results of any measurements of length always depend on the state of motion of the particular distances with respect to the system of reference chosen. Therefore, the length of a rigid body, too, is an entirely relative notion.

So far, your distance AB was lying exactly in the direction of the motion of our two systems O and O_1. If now you turn the line AB gradually out of this position, farther and farther—the two systems themselves continuing in the original direction of their motion—it will, as a matter of course, remain absolutely unchanged as to length, *as far as you are* concerned. *To me*, however, as compared with my distance A_1B_1, it will appear—as you can conclude immediately from Einstein's formulæ—less shortened the more it is turned away from its original position. In this case my measurements of your distance AB will differ less and less from your own statements, and, at the moment when the distance AB stands exactly perpendicular to the common direction of our two systems O and O_1, my measurement of your distance AB will exactly coincide with your own measurement. As soon as you turn your line AB farther beyond this position the results of our measurements will immediately differ again, and the more you turn it the more I shall find a constantly increasing contraction as compared with your statements. As soon as your distance AB returns

into its old direction my measurement will differ from yours again by the original amount. And, again, you will make entirely corresponding observations if I, in my system O_1, turn my line A_1B_1 out of its original position while you, from your system O, determine the length of my distance A_1B_1 as frequently as you can.

Our last results admit of a further deduction, exceedingly interesting, and at first sight extremely surprising. If you in your system O draw a circle round any point with a radius of any given length, I, from my system O_1, shall be unable to regard this figure as a circle. For, owing to my motion relatively to your system O, the distance of the centre from every point of the circumference will, as you know, appear to me to depend on the direction of the straight line joining the two points. In the direction of the motion it will appear shortest, perpendicular to it, longest, and in the intermediate positions I shall find a continuous gradation between these two values. Your circle will, consequently, appear as an ellipse to me, the short axis of which will lie in the direction of our mutual motion, while its long axis will stand perpendicular to this direction. Vice versa, *you from your system* O will take a figure for an ellipse which I, with reference to *my system*, have to accept as a circle. Analogous observations hold for all geometrical structures and material forms, so that we are compelled to regard the form or shape of bodies as relative conceptions, too, which only have a definite meaning when referred to a definite system of co-ordinates.

From Einstein's formulæ it appears, furthermore, that the degree of contraction with regard to the length of a distance is greater the greater the velocity with which the distance in question is moving with reference to the system of co-ordinates adopted. And it becomes evident that the length of a distance would shrink to nothing if it moved in its own direction with the velocity

of light. From this fact, as well as from others that cannot be explained here, we may draw the very important conclusion that, in accordance with Einstein's theory of relativity, the velocity of light represents a limiting value which can, in reality, never be reached, much less exceeded by any bodies.

When, for the first time, we were talking about the negative result of the Michelson-Morley experiment, I acquainted you with the attempt made by the Dutch physicist, Lorentz, to explain this fact by the hypothesis of contraction. As you will remember, this theory stipulated that every body moving through the æther at rest experienced a shortening in the direction of its motion. In Einstein's theory, too, we meet with this shortening. And it is a shortening of exactly the same magnitude as was claimed by Lorentz. Nevertheless, there is a fundamental difference between the two theories in this respect, too. For, according to Lorentz, the shortening is a fundamental property of matter, while in Einstein's theory it merely appears as a result of our methods of measuring distances. The relativity of our statements of distances is only caused by the nature of these methods, and thus we are relieved of the trouble of ascribing to matter special properties which might possibly explain its behaviour with regard to motion—a problem which Lorentz's theory must necessarily face.

As we have emphasised repeatedly in the course of our discussion so far, Einstein's formulæ were derived simply and solely in an endeavour to interpret, in a natural way, the result of the Michelson-Morley experiment, which remained unintelligible on the basis of previous conceptions. Now, if the usefulness of Einstein's formulæ were merely restricted to this one case, physical science would hardly have agreed to accept the heavy sacrifices with respect to old and familiar ideas which a recognition of Einstein's theory inevitably

carries with it. But, fortunately, this is by no means the case. On the contrary, there is quite a number of other phenomena by which the accuracy of Einstein's formulæ can be tested. However, since a more accurate understanding of these tests presupposes a comprehensive acquaintance with physics, and, moreover, remains unattainable without the use of mathematical methods, we will rather not give any details about them. You will simply have to content yourselves with the fact that the theoretical deductions resulting from Einstein's fundamental assumptions have been confirmed in the most brilliant fashion in all cases accessible to examination. If the mere fact that the principle of relativity can be tested by experiment is an immense advantage in itself, its importance is considerably enhanced by the other fact that the result of all these tests has been favourable to the principle of relativity.

Now, I am quite willing to believe that you can only slowly, extremely slowly, take in all the new and never-dreamt-of ideas which Einstein's theory of relativity thrusts upon you. It contradicts our customary ways of thinking in such a manner that the difficulties of grasping it appear insurmountable. It looks as if the gulf could not be bridged over. Our conception of the world is shaken to its foundations; the belief in one unmoving absolute space, this boundless "box" of æther, in which all that happens in the world was to take place, will have to go. We must also abandon the belief in an unchangeable true time, such as was ever accepted as the symbol of the flight of all phenomena. Distance in space, and duration in time, become meaningless conceptions, and the same holds with regard to the conceptions of rigid bodies and the simultaneity of two events. They all have to be relativised. They all receive a definite meaning only when the system of co-ordinates is given to which these conceptions are to be referred. Hermann Weyl has prettily illustrated

this state of things by a striking example. The life process of a human being may well be compared with the movement of a clock. Its course, therefore, will depend on the state of motion of the system of co-ordinates in which the particular man is spending his life. Now imagine twin brothers who, one day, take leave of each other. Let one of them stay at home, *i.e.* let him be permanently at rest in a suitable system of co-ordinates. But the other one is to go on a journey, and to travel with velocities as great as possible relatively to his home. If many years afterwards this traveller returned home he would find himself noticeably younger than his brother who remained at home.

IX

THE FOUR-DIMENSIONAL WORLD

FROM ancient times learned men in their wisdom have asserted that space represents a three-dimensional structure. It is usual to illustrate the meaning of this assertion by the additional statement that space possesses length, breadth, and height, as distinct from surfaces which have only length and breadth, and from lines which have merely length, or finally points, which have no magnitude at all. If we accept the conception of the mathematical point as given, we are able to regard space, surfaces, and lines as assemblages of an infinite number of points, the arrangement of which is regulated by different laws in the different cases. Thus, for instance, the points of a straight line are arranged according to a law essentially different from that governing the arrangement of the points in the circumference of a circle, and entirely different again from both is the arrangement of points on a surface—let us say the surface of an oval body. But all such assemblages (or manifolds) of points have one thing in common—if, from any point of a particular manifold, I pass to the adjacent point, then, by this transition, I never get outside the manifold. Each manifold consists of an uninterrupted succession of points. Each point is immediately adjacent to the preceding one, and there is no question of intervals or distances between any two neighbouring points. This exceedingly important pro-

perty of points has been known to you long enough—at least unconsciously. You are well aware, for instance, that a straight line is a continuous succession of an infinite number of points, and so you will, doubtlessly, understand now what the mathematician means when he calls such a straight line a continuum. Any other line, in this sense, is also a continuum, as also is every surface, and, finally, space itself. No doubt they are continua of entirely different nature, as you instinctively feel at once. For, as far as a line is concerned, from one point within the continuum you can only proceed in two directions—forward or backward—to reach another point, while the conditions with regard to a surface, and, most of all, with regard to space, are essentially different in this respect. Here not only two, but infinitely many possibilities present themselves, of getting from one point in the particular continuum to a neighbouring point in the same continuum. Lines evidently represent continua of the simplest kind, characterised exactly by the fact that constant progress within them is only possible in two directions, and the mathematician calls such continua—extending in one direction only—one-dimensional continua. Any particular line, therefore, is to be regarded as a one-dimensional continuum. You need now no longer be afraid of this word that sounds so awfully learned, for you see that, after all, it has a very simple meaning. It is merely for the sake of convenience that we are going to use the word in future. For, after once you have grasped its meaning, this one word will be sufficient for you to recall to your mind all our previous explanations. So we need not repeat these discussions later on; they lie, as it were, encased in that one word, like a mighty rose in its insignificantly small bud. And if, further on, I make use of mathematical terms, I do so in each case for no other reason than to summarise in a brief word the result of tedious explanations in order to enable us

to recall them to our minds as conveniently as possible for future purposes.

After having expressed the peculiarities of the manifolds of points in a line by the term "one-dimensional continuum," we are able, without much trouble, to give a summary of the other continua as represented by surfaces and space. First of all, imagine a plane represented, let us say, by the surface of a table. You could conceive this plane, too, as a one-dimensional continuum. Only, in that case, you ought not to regard points as the constituent elements of the continuum, but you would have to conceive the plane as a manifold of an infinite number of lines which, just like the points in a line, are situated side by side in a continuous manner, *i.e.* without intervals. Regarded in this way, each plane actually constitutes a one-dimensional continuum of lines, for, as a matter of fact, you can proceed from any particular line of the continuum to an adjacent line in two directions only, moving either forward or backward. However, if we now look upon the plane as a continuum not of lines, but of points, we feel inclined to call the plane a two-dimensional continuum. Simply because it represents a one-dimensional continuum of what are themselves one-dimensional continua. And this consideration not only applies to planes, but we can easily transfer it to all other kinds of surfaces. One simple example should be sufficient to make this clear to you. As you are aware, the circumference of a circle—being a line—is a one-dimensional continuum. Now imagine an infinite number of circular lines absolutely alike and placed side by side without a gap, in the same way as we can push several cartwheels on one and the same axle. A surface would evidently result—in this case the surface of a cylinder. Consequently we have a right to regard such a cylindrical surface as a one-dimensional continuum, consisting of continua that are one-dimensional in them-

selves, so that the result is a two-dimensional continuum. If the circles used by us for the production of a surface had not all been of exactly the same size, but if each circle chosen had been smaller than the circle immediately preceding it by an infinitely small amount, the result would have been a semispherical, not a cylindrical surface. Again, therefore, we may call the surface of a sphere a two-dimensional continuum. Something analogous applies to all surfaces, and thus we are led to calling any particular surface a two-dimensional continuum.

By now you will, no doubt, be able to foresee the course our further considerations are likely to take. If, for instance, we are considering a cube, there is nothing to prevent us from imagining that this cube was built up by continuously placing side by side an infinite number of squares. A cube is nothing but a part of space bounded in a definite manner, and a square is nothing but a part of a plane bounded in a definite manner. We may, therefore, give expression to our mental experiment described just now, by saying that we are able to produce a bounded part of space by continuously placing side by side an infinite number of bounded parts of a surface. In a similar way we could construct a cylinder out of equal circles, or a sphere from an infinite succession of circles of diminishing size. In a word, any particular bounded part of space can be constructed out of definitely bounded parts of surfaces. And, finally, we may imagine that limitless space, too, was produced by putting together an infinite number of limitless surfaces. As far as the final result is concerned, it is a matter of complete indifference whether, for this purpose, we choose plane surfaces or surfaces curved in any particular way. Boundless space will result from a continuous succession of infinitely large planes as well as from an infinite succession of constantly increasing spherical surfaces fitted together like the skins of an

onion. Thus we are entitled to regard space as a one-dimensional continuum of surfaces. But since each surface represents a two-dimensional continuum, we shall be consistent if we regard space as a three-dimensional continuum.

Now, if we try to take in, at a glance, the whole of the one-dimensional continua (*i.e.* lines), a remarkable difference between them will soon occur to us. We can illustrate this difference most clearly by assuming, for a moment, that the world itself is only a one-dimensional continuum. In such a world there could be merely one single straight line, *i.e.* the world itself. There would be no room in such a world for the large number of curved lines. For what do we mean by calling a line "curved"? We mean nothing more nor less than that, exactly like a straight line, it constitutes a one-dimensional continuum but that, for its development, it involves the existence of a second, if not of a third dimension. The circumference of a circle lies, one might say, embedded in a particular plane; but the wire of a spiral spring follows a line drawn through three-dimensional space. Although one-dimensional in themselves, both lines can, therefore, only exist in a world of several dimensions. If the world were two-dimensional, the lines of spiral springs would be as impossible in it as spherical and cylindrical surfaces, and as all surfaces deviating from the plane, *i.e.* surfaces curved in any way whatsoever.

Our space is, doubtlessly, a three-dimensional continuum. In our previous investigations we have tried to illustrate this space by a system of co-ordinates in space which enabled us at the same time to determine the position of points in space, *i.e.* to carry out measurements in space. Every individual point was described by its three co-ordinates, by which were meant its distances from those planes of the system chosen. We could reach the same object in a slightly different way.

Let us briefly call the three respective planes of the system the U-plane, the V-plane, and the W-plane. Their common point of intersection indicates a very definite point in space, the so-called origin of the system. But we may also regard any other point of space as a point of intersection of three planes if, parallel to the three planes of the system, we draw a large number of other planes, every one of them having the same distance from the one immediately adjoining it. Let us choose this distance as minute as possible, and designate its length as unit. Each of the planes parallel to the U-plane is equally to be called a U-plane. But, in order to be able conveniently to distinguish from each other the infinite number of U-planes, let us designate the original U-plane of our system as "U-plane zero," the one immediately adjoining it as "U-plane 1," the one following next as "U-plane 2," and so on, until finally all the U-planes are numbered consecutively. We then do the same with the V-planes and the W-planes. As you will easily see, the total number of all the U-, V-, and W-planes will divide boundless space into an infinite number of small part-spaces shaped like cubes. And if we make this division as fine as possible by choosing a sufficiently small distance between two parallel planes—*i.e.* by adopting a sufficiently small unit—we shall actually be in a position to regard any particular point as the intersecting point of three particular planes. Its position will then be described by three figures indicating the numbers of the U-, V-, and W-planes, intersecting each other in that particular point, and which we have a perfect right to call the co-ordinates of that particular point in the system of co-ordinates chosen.

I am sure you will not doubt that the method described just now is admissible. In all probability you will even feel inclined to assume that it is the only justifiable method of determining the positions of points

in space unequivocally by means of three systems of surfaces. The possibility of dividing space in this manner, however, presupposes the validity of the so-called Euclidean geometry, *i.e.* of that geometry which was built up on the three axioms previously stated, and as whose founder we may regard Euclid, the most important mathematician of ancient Greece. An expert will at once notice the connection of the method above with those three axioms. You will see it is the best possible way if you consider separately one of the cube-shaped part-spaces obtained.

In order to make it possible for six absolutely equal cubes to attach themselves to the six limiting surfaces of the small cube—this being the essential point with respect to our method—every one of these surfaces must be a square, and every angle of each square a right angle. The sum of the four angles of each single square must, in consequence, necessarily amount to four right angles. This proposition can easily be proved by means of elementary school geometry, *i.e.* its truth can be reduced to the truth of the geometrical axioms. Only if the three axioms of geometry are true will our assumptions with regard to the angles in the square be correct. And only in this case can the method of dividing space in the manner described be carried out without leaving anything over, and without difficulty. For this reason it is quite fitting to say that space must possess Euclidean structure to make the application of our method possible. But the assumption of the Euclidean structure of space is not the only possible one. This follows with striking clearness from the following example, which we have borrowed from the considerations of Poincaré, the French mathematician, in his book *Science and Hypothesis*.

Imagine a world enclosed in a large sphere, inside which the distribution of temperature is quite irregular. Let the temperature be highest at the centre of the sphere;

let it grow less and less in all directions, until at the surface of the sphere it reaches its lowest possible value. As you know, all bodies expand on heating, and contract on cooling. Let us now make the further assumption that all bodies in this imaginary world behave absolutely alike with regard to the influence of temperature. For instance, a measuring-rod is always to expand by the same amount for a particular rise in temperature, no matter of which material it has been made. And, finally, we will assume that each body, when it moves inside the sphere, always *immediately* suffers those changes of form which correspond to the changes of temperature through which it passes.

Now I should like to ask you if an inhabitant of such a world would be able to construct a threefold number of planes across his space in the manner described above? No doubt he would not be. In order to see this you only have to think of the behaviour of bodies heated unevenly. Take a sheet of gelatine in your hand by the edge. Owing to the heat of your body it will get warm at the place where you touch it. This higher temperature gradually spreads to the neighbouring parts; and, as a result of the unequal heating thus produced, you notice a distinct curvature of the whole sheet. What in this case happened to the gelatine will happen to the planes in the world described above. They will all get curved in a manner depending on the way in which the temperature is distributed inside the sphere. The U-, V-, and *W-planes* of the previous system of co-ordinates will, as a result, be turned into curved *surfaces* of arbitrary curvature. The small part-spaces which constitute the whole of space are no longer cubes, but some sort of eight-cornered space structures, all differing from each other. Each part-space is bounded by six curved part-surfaces, none of which is equal to the other. And each particular surface contains four angles different from each other, and whose sum does not amount to

four right angles. In other words, in a world of this description one is bound to arrive at a so-called non-Euclidean geometry which completely deviates from the school geometry with which we are familiar. In spite of all this, however, the determination of position in space is still possible by means of a system of co-ordinates. Only the fundamental constituents of such a system of co-ordinates would, obviously, not be planes, but those U-, V-, and W-surfaces whose shape depends on the distribution of heat and cold inside the sphere. We can assign to them consecutive numbers, just as we did to the planes before, and each point will again be described by three numbers which we have a perfect right to call its co-ordinates. They are called Gaussian co-ordinates, because they were introduced into science by the mathematician Gauss.

You will easily see that these Gaussian co-ordinates offer essential advantages in comparison with the co-ordinates used previously. Formerly we meant to determine the position of a point in space by measuring its distances from the three planes of a system of co-ordinates. In a space of non-Euclidean structure, such as we met with in the example given above, that method becomes utterly meaningless. For what would we have to understand by the distance of a point from the planes of a system of co-ordinates in an unevenly heated sphere? We should occasionally get lines twisting about like an adder! If, on the other hand, we indicate the positions of the points in space by the numbers of the U-, V-, and W- surfaces which intersect at a particular point, this method will remain applicable to spaces of any geometrical structure whatsoever. If space has a Euclidean structure all of those surfaces will be planes. If it has not, the surfaces will show curvature of some sort, possibly even extraordinarily complicated ones; but it will always be possible to define in the manner described

the positions of all points in space quite definitely by means of these surfaces.

But now we come to the most remarkable point. So far, you have regarded the world enclosed in that sphere from our generally accepted point of view. This means that you looked at it fully convinced of having a clear insight into its conditions. Above all, you recognised the structure of its space as deviating from Euclidean structure. But what is going to happen if you are suddenly transplanted from our familiar world into another one, lying embedded in a space of non-Euclidean structure? If you were to become an inhabitant of our imaginary world inside the sphere? The answer is self-evident. On the assumptions we have made, you would . . . not notice anything of the irregularities actually existing! You are rather taken by surprise, and yet you cannot help agreeing with this conclusion. For how would you be able to establish the deviations from cubic form undergone by those frequently mentioned part-spaces, except by measuring them up with a standard measuring-rod? This measuring-rod, however, will adapt itself exactly to the local conditions of temperature. It will be curved like all the other objects of that world, and you will never become aware of the fact. Because your body, too, with all its organs, is bound to become curved in a similar manner, and you will be without the possibility of comparing the new state of things with the former one. Everything has at once adapted itself perfectly to the new order of things. All changes, therefore, however fundamental they may be, will always remain hidden from you, and all the time you will imagine you are studying Euclidean geometry, whereas, in reality, you are devoting yourself to non-Euclidean geometry.

This last conclusion of ours necessitates at once a further one. If you are really constrained to assume that you are doing Euclidean geometry in that imaginary

world—what is there to constrain us to consider our present geometry as a truly Euclidean one? Would it not be quite conceivable that, for some reason or other, our space possesses a non-Euclidean structure, but that we have no knowledge of this fact, and consequently we base our geometry on the assumption of a Euclidean structure? That, in reality, a "cube" is not a cube, but a part of space bounded by completely twisted surfaces? That, in reality, a "cube" does not remain unchanged when I take it from one place to another; but, on the contrary, it either expands or shrinks in the most complicated manner according to the conditions existing in that space? All this would be possible, and there is nobody in the world who could prove that we are on the right track with regard to our present geometry! This whole geometry, with its high and proud structure of ideas, would be nothing but an entirely arbitrary assumption justified by no facts whatever.

An assumption? Of this there can be no doubt; but an assumption suggested to us by experience. In nature we actually meet with rigid bodies which preserve their form unchanged when they are moved about. By them we were led to our general geometrical conceptions, such as body, surface, and line. The laws governing physical phenomena can be expressed in the most simple way if, in describing them, one adopts Euclidean geometry. You only have to think of the law of inertia, the law of the rectilinear propagation of light, and of Kepler's laws regarding planetary motions, in order to understand how this is meant. No doubt it would be possible also to describe all these phenomena quite definitely if one ascribed to space any non-Euclidean structure. Only, in this case, one would arrive at extraordinarily complicated relations. For exactly in the same way as, from our Euclidean point of view, the world in the unevenly heated sphere appears to us completely distorted, so our observations would have to

undergo an essentially different interpretation if we based them on a non-Euclidean structure of space. The form of bodies would depend on the place where they are situated, consequently it would change more or less considerably as a result of motion. Rays of light would travel in curved lines. A motion which obeyed the law of inertia would follow an irregularly curved course. And the courses of the planets round the sun would be complicated to such an extent that the simple relations we find expressed in Kepler's laws would be absolutely impossible. Therefore, as long as there are no absolutely imperative reasons why we should depart from the familiar Euclidean conception, we shall certainly shrink from taking such a step. Nevertheless, it remains conceivable "that the measure relations of space are not in accordance with the assumptions of our geometry, and, in fact, we should have to assume that they are not if, by doing so, we should ever be enabled to explain phenomena in a more simple way."

These last words are a quotation taken from a famous inaugural address, "The Hypotheses Underlying Geometry," delivered by the learned German mathematician, B. Riemann, on the 10th June 1854, on the occasion of his appointment as a member of the Philosophical Faculty in the University of Göttingen. These words express most clearly the fact that we have simply no right to speak of a structure of "space in itself." In themselves all non-Euclidean geometries have the same justification as Euclidean geometry. Only for the sake of convenience we make a definite choice so as to be able to describe all natural phenomena as simply as possible.

In such a description of natural phenomena space always appears in closest relation to time. The objects and events which we perceive represent, in each particular case, nothing but combinations of places in space and time. Never have you met with a definite place, except

at a definite time, and only at a definite place have you ever noticed a definite time. Space and time appear indissolubly intertwined. Starting from the recognition of this fact, the German mathematician Hermann Minkowski felt constrained to introduce a fundamental change in the generally accepted ideas about space and time. "From this time forth space in itself and time in itself are to become mere shadows, and only a sort of union between the two is to preserve independence." Thus spoke Minkowski in a lecture delivered at Cologne, on the 21st September 1908, at the 80th Conference of German Scientists and Medical Men. The world of physical phenomena is to be regarded as a four-dimensional continuum. You will easily understand that this way of looking at things is justified. Take the simplest instance of a physical "event," a material point at rest in three-dimensional space. The point is, at each moment, in one and the same position of space, and the "event" is therefore nothing but a continuous succession of one and the same space-point in the constantly onflowing time. Exactly corresponding to this is our former consideration, according to which, for instance, a cube is a continuous succession of one and the same plane square surface in a constantly increasing "height." And since, furthermore, a sphere may be regarded as a continuous succession of different spherical surfaces, encased one within the other, the event of the motion of a material point in three-dimensional space represents a continuous succession of different places in space. In other words, every particular physical event is to be regarded as a continuous succession of space-points in time. Thus it seems quite appropriate to look upon the world of these events as a four-dimensional continuum, the elements of which are an infinite number of "event-points," or "space-time-points." Minkowski, in a

terminology easily grasped, calls each point of this kind a "world-point"; their total number constitutes the "world." The mathematician will thus be able, by appropriate adaptation, to transfer to the four-dimensional world the geometrical methods used for the investigation of three-dimensional space. In this world the position of each world-point is evidently described by four co-ordinates — three-space co-ordinates to indicate the space-point of any particular event, and a one-time co-ordinate to indicate its time-point. The customary three-dimensional geometry thus passes into a four-dimensional "system of events" or physics. To give you a clearer idea, we had better quote a few sentences from Minkowski's address, only slightly altered in a few places to make it easier for you to understand. "By taking a piece of chalk I could boldly throw on the blackboard the four world-axes of a system of world-co-ordinates. One axis alone, when drawn on the blackboard, would consist of numerous vibrating molecules, and since it would be travelling with the earth through the universe it would be in itself a sufficiently difficult task in abstraction; the slightly greater abstraction connected with the number 4 does not hurt a mathematician. In order not to leave a yawning emptiness anywhere, we shall imagine that something perceptible does exist at every place and at every time. To avoid the word matter or electricity I'll call this something by the name of substance. We will turn our attention to the substantial point existing at a particular world-point, imagining that we shall be able to recognise this substantial point at any other time. Let quite definite changes of the space-co-ordinates of this substantial point correspond to a definite small period of time. We obtain thereby, as a graphic representation, so to speak, of the eternal career of this point a curved line in the world, a world-line, the individual points of which can be referred to all time-values

possible. The whole world will appear dissolved into such world-lines. . . . We shall then, in this world, have no longer *the* space, but an infinite number of spaces just as there is an infinite number of places in three-dimensional space. Three-dimensional geometry will become a chapter of four-dimensional physics. You will see why I said at the beginning—space and time are to become shadows, and only a world in itself is to exist."

But what is the good of all this you will now doubtlessly ask? Why this monstrous step directed towards a generalisation of our conceptions of space? It may amuse a mathematician to plunge into matters of this kind. But why should this trouble the physicist?

Well, it is just for the physicist that the idea of a four-dimensional world offers immense advantages. It leads to a simplification of our conception of the world, such as no one has ever dreamt of. For in the further pursuit of the idea established by him Minkowski arrived at the surprising result that *the validity of Einstein's special principle of relativity means nothing more nor less than this, that the four-dimensional world possesses a Euclidean structure.* In other words, exactly the same method by means of which in *Euclidean* three-dimensional space we are able to describe unequivocally the position of points, may be extended, if suitably generalised, to the four-dimensional world as well. If we have before us any physical event, let us say the motion of any particular body in our former method of consideration in which space and time were treated *separately*, we had to transform the measurements of space and time according to Einstein's formulæ in the transition from one particular system of co-ordinates to another in uniform rectilinear motion relatively to the former. But if we consider that event as a "world-line" in the four-

dimensional world the remarkable fact results from Minkowski's investigations that a "rotation" of the four-dimensional world-system corresponds to that transition from a three-dimensional system of co-ordinates to a second one. After this "rotation" the particular world-line will, naturally, assume a different position with reference to the four-dimensional world-system. Its mathematical description will, therefore, look different from what it was before. And, as was shown by Minkowski, the relation between the former and latter description is exactly expressed by Einstein's formulæ, in which the special principle of relativity finds its expression.

X

EINSTEIN'S GENERAL PRINCIPLE OF RELATIVITY

ONCE more recall briefly the method which we adopted in formulating the special principle of relativity. By the Michelson-Morley experiment, with regard to the phenomena connected with the propagation of light, the equivalence of two systems moving, relatively to each other, uniformly and in a straight line, had been discovered. For the equivalence of these systems we then claimed *general validity*. As you know, this postulate could be fulfilled, on the whole, by a relativisation of the conception of time.

Now the statement contained in the special principle of relativity assigns to systems of co-ordinates in uniform rectilinear motion an amazingly privileged position in nature. Why should this one class of systems be distinguished in such an outstanding manner from the much larger number of systems moving non-uniformly in curved lines? Involuntarily this question will occur to a thinking man, and his desire for knowledge will urge him towards a generalisation of the idea of relativity. Not only with regard to systems in uniform rectilinear motion, but with regard to *all possible systems of co-ordinates* the laws of nature ought to be fulfilled, so that the conception of *absolute* motion would become perfectly meaningless. This, however, is entirely contradicted by our experience. It is a fact that, in the interior of a tramcar travelling with uniform

rectilinear motion, you are unable to discover this motion by any physical experiments whatsoever, if you refer all your measurements to a system which is at rest with reference to the car. As soon, however, as the driver applies the brakes, and the motion of the car is suddenly *slowed down*, unmistakable deviations from the laws of nature will come into existence. Your body, which up to that moment was at rest in the car, experiences a noticeable impulse forwards, so that the law of inertia, which made you expect that your body would be *permanently* at rest with reference to the car, is no longer fulfilled. Moreover, as long as you perform experiments of sufficiently short duration on the surface of our earth—let us say experiments with rays of light—during which the motion of the earth may be regarded as uniform and rectilinear, not the slightest influence of this motion on the progress of the experiments will be traceable. But if you spread your observations over a longer period—as, for instance, in the case of experiments with the pendulum—at once an unmistakable deviation from the laws of nature will appear, caused by the curvilinear motion of the earth.

Hence, if we try to do justice to the postulate of a *general* principle of relativity, we shall have to introduce radical changes into our customary conceptions. Before doing so, however, let us carefully examine whether, in all circumstances, we actually are in a position to recognise infallibly a non-uniform motion of our system of reference as such.

For this purpose we will make use of an illustration given by Einstein. Imagine a spacious chest, shaped like a room, in which we are enclosed with all the apparatus required for accurate physical measurements. Let this chest be at rest relatively to the world-space. Let it be so far away from all heavenly bodies that not the slightest noticeable trace of attraction acts upon it. In this case there will naturally be no force of gravity.

All the objects in our room will be absolutely without weight. For you know well enough that the weight of bodies on our earth is nothing but an effect produced by the gravitational force of the earth. Now, while we are in that chest, let a distinct pull suddenly become noticeable, which strives to pull us towards the floor, and owing to which all the objects in our company begin moving in a corresponding way. Let them all fall towards the floor with constantly increasing velocity. How should we interpret these observations?

Two possibilities present themselves. First of all the assumption that our room, relatively to space, was started on a constantly accelerated motion, *i.e.* a motion constantly increasing its velocity, directed upwards. In that case we could easily explain why our bodies, together with all our apparatus, were being attracted towards the floor of the room. For, since up to that moment they were at rest with reference to space, they had a tendency, according to the law of inertia, of remaining unchanged in their state of rest for all time. Thus they persisted in their original position of rest when the chest entered upon a state of accelerated motion. Consequently the room, relatively to space, rushed towards its contents; and the objects, relatively to the room, had, in a corresponding manner, to move downwards with increasing velocity. If they were prevented from falling by supports standing on the floor of the room, they had to exert pressure on these supports, and the supports, in their turn, had to transmit this pressure to the floor of the room. In this way all our observations become immediately intelligible.

On the other hand, we might just as well draw the following conclusions: our chest continued at rest all the time; but immediately underneath a heavenly body suddenly appeared which, by its gravitational force, acted upon the contents of our room. Consequently all our instruments, together with ourselves, were

attracted towards the floor. This interpretation, too, can be carried through without inconsistency, and a decision in favour of either the one or the other will be absolutely impossible. Therefore both have to be regarded as entirely equivalent.

That even the most accurate physical measurements are unable to alter the equivalence of the two possibilities of interpretation suggested by the example just mentioned is due to an empirical fact of extraordinary importance. I am referring to the fact that, under the influence of the gravitational force of the earth, a freely movable body enters into an accelerated motion, and that the velocity of the fall is exactly the same for all bodies, no matter of what material they are composed. Everyday experience, no doubt, seems to prove the contrary. For, if you let a piece of paper and a leaden ball drop simultaneously from your hand, the leaden ball will obviously fall considerably faster than the paper. This, however, is merely due to the fact that the falling motion takes place through the air. On account of its greater weight lead is able to overcome the resistance of the air more easily than paper. Therefore, to obtain the phenomena of fall in their pure form, one has to perform the experiments in a space void of air. Then it will become evident, indeed, that not only pieces of lead or paper, but of any material, will fall to the ground with equal velocity. Now, the fall of bodies is simply the best known instance of the effect of gravitation. In a preceding chapter we spoke, as you know, of Newton's laws of gravitation, according to which each body possesses gravitation, *i.e.* a faculty of exerting some force of attraction on all other bodies. One imagines the matter to be like this: by every body space is thrown into a peculiar state, which physicists call a gravitational field. The nearer a body approaches, the more the intensity of the gravitational field will increase, and the gravitational field, by laying hold of

all bodies situated within it, and by striving to pull them towards the body which produces the field, causes them to move faster and faster the nearer they approach to the body attracting them. From the fact that all bodies on the earth fall with the same velocity, we may draw the further conclusion that each gravitational field gives an acceleration of exactly the same magnitude to all bodies, regardless of the material of which these bodies are composed.

You will now easily understand that the possibility of giving a twofold interpretation to the observations made in our chest is exclusively due to this circumstance. For if gravitation had a preference for certain kinds of matter, we should have no right whatever to ascribe the sudden uniform falling motion of all our instruments to the influence of a gravitational field. On the contrary, no other assumption would then be left to us than that our room was at rest up to the moment when, for some unknown reason, it began to move, assuming a constantly increasing velocity, whilst the bodies, owing to their inertia, produced the illusion of a motion in the opposite direction—in this case a falling motion. Now, however, we are in the unpleasant position of being able to regard, with the same amount of justification, one and the same phenomenon as the effect of inertia or the effect of a gravitational field. The fact contained in this statement is called "principle of equivalence" by Einstein. According to it we are thus unable to prove with certainty the fact that our chest is starting on an accelerated motion.

As soon as we recognise this the objections which we raised with respect to a generalisation of the principle of relativity necessarily begin to waver. For we have an equal right, in the example chosen above, to refer our observations to a system of co-ordinates at rest relatively to world-space, or to one in accelerated motion. By raising the principle of equivalence—which

hitherto we have only found confirmed in the one case quoted—to the position of a principle having general validity, we may now formulate the general principle of relativity in the following provisional manner: all systems of co-ordinates which, relatively to each other, possess a motion of any kind, are equivalent as regards the description of natural phenomena.

Without a doubt this much is already certain—the recognition of the general principle of relativity will require a new formulation of the laws of nature. In their present form they only hold, as you know, with respect to systems of co-ordinates in uniform rectilinear motion. About this alteration of the laws of nature we shall have to speak again later on. First of all we shall look at the consequences resulting from the general principle of relativity with regard to measurements of space and time. Here, again, it will serve our purpose best if we start from an example suggested by Einstein.

Imagine a flat circular disc which, over a plane "at rest," is uniformly rotating like a roundabout on an axle passing vertically through its centre. A physicist who happens to be on this disc will everywhere, except at the centre, feel the effect of some force which is trying to fling him off the disc, and which grows stronger the nearer he approaches to the edge of his disc. This force is familiar to you by the name of centrifugal force, and you are used to regard it as the result of rotary motion relatively to the "plane at rest." Our physicist, however, knows nothing of this motion. He takes his disc to be at rest, and attributes the effects of force appearing everywhere to the influence of a gravitational field, the intensity of which varies from place to place.

Now, will this physicist be able to draw a square of four straight lines of the same length? Certainly not; because on his rotating disc even the conception of a straight line becomes perfectly meaningless. You will

understand this easily by just thinking about the results arising from the special principle of relativity. Apart from the centre, all points of the disc are, as you know, in motion with reference to the plane "at rest." And, generally speaking, the velocity of this motion varies everywhere on the disc. Close to the centre it is comparatively small; it increases with the distance of any particular point from the centre. In consequence, according to Einstein's formulæ, to which the special principle of relativity has led us, a line will suffer a definite contraction everywhere on the disc, and this contraction will vary from point to point. Now think of a stick of wax which you heat slightly at one end. The heat passes along the stick, but so slowly that to each part of the stick a different temperature is imparted. Owing to a rise in temperature wax expands—just like all other bodies—and it will do so all the more the greater the rise in temperature. Our stick, therefore, will expand at a *varying rate* in all its parts, and, as you know from experience, this becomes apparent at once in a bending of the whole stick. Something exactly similar will happen to a straight line on the disc described. In this case, however, the varying length of the individual parts of the line is not brought about by temperature, but by the varying velocities of motion with reference to the plane "at rest." But the *reasons* for the differences in length are matters of indifference; the main point is that the differences *do exist*. And just as the stick of wax in the instance just given will bend owing to the varying length of its parts, so a straight line on the disc of our physicist will become curved. Generally speaking, the degree of curvature will differ according to the different positions of the line. The conception of the "length" of a line thus loses all its meaning, since its amount will vary constantly for each change of place. The same will obviously apply to "distance" between two points of the disc. Thus the

construction of a square will become impossible to the physicist. He will always obtain pieces of surfaces circumscribed by curved lines; and least of all will he be able to place side by side squares of equal size.

Perhaps we shall get a still better idea of the state of things if we take into consideration the m e a s u r e m e n t s o f t i m e made by our physicist. Let us assume that, at several points of his disc, he places identically constructed clocks, *i.e.* clocks which go at the same uniform rate on the plane "at rest." No trace, however, of such a uniform "rate of going" will be noticeable on the rotating disc. For in general each clock has a motion of ever-varying velocity with reference to all the other clocks. The farther away from the centre of the disc the clocks are, the slower they will go, and only clocks that are equidistant from the centre will have the same velocity. As we know from our considerations on the special principle of relativity, clocks will go the more slowly the faster they are being moved. So we shall have to conclude that all clocks on the rotating disc will go at different rates, and that, as a result, the conception of time can no longer be defined. We can, then, no longer say what we mean by the "interval of time" between two events, because this "interval" will always be judged differently if we use different clocks o n t h e s a m e d i s c.

Owing to this rather unpleasant fact we find ourselves face to face with a difficulty which we never met, either in classical mechanics or in the special theory of relativity. There it had always been possible for us, without any trouble, to assign a consistent physical meaning to our measurements of space and time. If, for instance, we said that the measure of a distance amounted to ten kilometres, we meant if, beginning at one end of this particular distance I place a measuring-rod, one kilometre long, repeatedly along the distance until I exactly reach the other end, it will be necessary

to apply the rod ten times. No doubt the special theory of relativity had taught us the fact, unknown to classical mechanics, that any measurement of this kind only had a meaning when referred to one definite system of co-ordinates. But within one and the same system of co-ordinates, according to the special theory of relativity also, the length of one and the same line was bound to be the same everywhere. The same applies to measurements of time. If we talked of a process, the duration of which lasted ten seconds, this statement meant that a perfectly constructed seconds-clock would complete exactly ten full oscillations from the beginning to the end of that process. In this case, too, the special principle of relativity in no way modified the assumption that, with reference to a particular system of co-ordinates, our judgment with respect to the duration of an event happening within the system is independent of place.

But things are quite different on the rotating disc. The conceptions of the "distance" between two points, and the "duration" of an event can here no longer be defined in a manner applying generally. On the contrary, space and time lose their "last remnant of physical reality," to use Einstein's own words. Now, in accordance with the principle of equivalence, we may imagine the rotating disc replaced by a disc at rest, on which there is a gravitational field of everywhere varying intensity. Consequently we are at once enabled to generalise our important result by saying that, within gravitational fields, the customary co-ordinates of space and time become meaningless conceptions. These words, however, mean nothing else than that the world of physical events, which we may now again regard, with Minkowski, as a four-dimensional continuum, possesses a non-Euclidean structure in the presence of gravitational fields. For its description we shall, therefore, have to use Gaussian systems of co-ordinates. For this reason

we will now express Einstein's general principle of relativity in the following final form: For the formulation of general laws of nature all four-dimensional Gaussian systems of co-ordinates are absolutely equivalent.

The space-time structure of the world is determined by gravitational fields. As you know, gravitational fields are produced by masses. Hence the space-time structure of the world will be "dependent" on the manner in which masses are distributed within it. From our previous considerations you know well enough what is understood by this "dependence." The particular methods of measuring the space-time continuum we shall have to adopt in order to be able to find a simpler interpretation of natural phenomena will depend, in each case, on the manner in which masses are distributed. Thus not only space and time, but space, time, and matter are indissolubly interwoven, and the words quoted previously from Minkowski's address may be amplified, without violating their meaning, in the following way: "From this time on space in itself, time in itself, and matter in itself are to become mere shadows, and only a sort of union between the three is to preserve independent existence."

Let us now consider the relations existing between the general principle of relativity and the earlier special principle of relativity. The latter had maintained that for the description of natural phenomena we have an equal right to use either of two systems of co-ordinates in uniform rectilinear motion relatively to each other. Only in our transition from one system to the other have we to transform our measurements of space and time by means of Einstein's formulæ. Or, to express the same meaning in other words, the four-dimensional space-time continuum possesses Euclidean structure, and the formulation of natural laws remains unchanged when a "rotation" is applied to the world system used.

On the other hand, the general principle of relativity postulates: for the description of all natural phenomena we are free to use, with equal right, any Gaussian system of co-ordinates we like. Only in our transition from the one system to the other we have to assign to the four-dimensional space-time continuum a structure modified in a manner accurately to be indicated. Now, since Euclidean structure may be regarded as a special case of a non-Euclidean structure, the general principle of relativity will have to pass over into the special one as soon as there is no trace of a gravitational field. The reason for this is that a gravitational field causes the deviations from Euclidean structure. But a complete absence of gravitational fields is unlikely anywhere, since the whole universe seems permeated by heavenly bodies, the gravitational fields of which extend to immense distances. In particular, on the earth we are undoubtedly within a gravitational field, so that the special principle of relativity can never claim strict validity with us. It is a law of approximation which may be used advantageously wherever, as is the case with our earth, only extremely weak gravitational fields have to be considered. Considered as a law of approximation, the special principle of relativity will retain permanent importance—quite apart from the fact that it has led to the discovery of the *strictly valid* general principle of relativity.

As we have emphasised before, the formulation of the laws of nature will have to be altered so that they will hold, according to the postulate of the general principle of relativity, with respect to all Gaussian systems of co-ordinates. Let us consider, for instance, the law of inertia in classical mechanics, according to which a material point, not influenced by forces, will constantly go on moving uniformly and in a straight line. Expressed in these terms, its validity is restricted to a case where gravitational fields are completely absent,

and, consequently, an ordinary system of co-ordinates is used. We have, therefore, to find a new formulation which, at one and the same time, covers the effects of gravitation, and, in the special cases mentioned, passes over into its old form. The solution of this problem became possible by amplifying, in a manner suitable for four-dimensional non-Euclidean continua, the definition of a straight line as the shortest connection in a three-dimensional Euclidean continuum. Such a shortest line in a non-Euclidean continuum is generally called a geodetic line. Einstein's fundamental law will then be worded in the following manner: The world-line of a material point is a geodetic line in the four-dimensional space-time continuum. This law does, indeed, satisfy all demands raised by us a short time ago with regard to a suitable fundamental line. For as soon as gravitational fields are absent, the space-time continuum will become Euclidean, and the geodetic line will be a straight line. Consequently, in this case, the new fundamental law will pass over into the law of inertia of classical mechanics.

The most brilliant triumph of the general principle of relativity consists in the fact that it leads to a theory of gravitation. For in our transition from one system of Gaussian co-ordinates to another, the mathematical expression describing the geodetic line changes in a very definite manner. Now, since a very definite change of a gravitational field corresponds to such a transition, you will understand that it should be possible to establish relations between these two changes by means of mathematical investigations of such a kind that they would allow us to find the laws governing the gravitational field. As a result, Einstein obtained a theory of gravitation, the sublime beauty of which discloses itself to the mathematician alone, whilst we shall have to be satisfied with having some of its consequences pointed out to us. Newton's law of gravitation which, previously, we had

come to regard as the law of the universe, loses its general validity; we have only a right to apply it when we are dealing with rather weak gravitational fields. Kepler's laws of planetary motion may, as you know, be derived as mathematical consequences following from Newton's law of gravitation. So one feels inclined to assume that these laws will no longer hold good with complete accuracy in cases where strong gravitational fields come into play. The deviations to be expected can be calculated on the basis of Einstein's theory of gravitation. Unfortunately they are, on the whole, so small that, with the resources at our disposal at the present time, we cannot yet think of seeing them confirmed by astronomical observations. There is only one exception in the case of Mercury, the planet nearest to the sun, where the effect is bound to reach a noticeable magnitude, since the planet's motion takes place in a comparatively strong gravitational field.

As a matter of fact, in this case the theoretical deduction is confirmed by experience in a truly startling manner. It has been known for a long time that the orbit of Mercury does not occupy in space a stationary position with regard to the fixed stars, but that it shows an extraordinarily slow rotation in the direction of the planet's motion round the sun. Or, to put it more clearly, that point of the orbit which is nearest to the sun, the so-called perihelion, in the course of time is shifting its position with regard to the world-space. This perihelial movement of Mercury flatly contradicts the theory held up to now. But from Einstein's theory its existence not only follows, but the numerical value of its velocity agrees most accurately with observation.

If this achievement obtained by the general theory of relativity may be regarded as a first valuable testimony to its physical correctness, other evidence is not lacking to show that its postulates hold in nature. I refer here to the fact that it proves the existence of influences

which strong gravitational fields have upon the propagation of light. First of all we find that the spectral lines of sunlight must be slightly displaced towards the red as compared with the spectral lines emitted by sources of light on the earth—a deduction which was confirmed by experiments. Still, we will not enter upon it in detail, because too much knowledge of physics is required for its full understanding. On the other hand, it will be easier for you to understand a second deduction, *i.e.* the deduction that light in a gravitational field will generally be propagated along a curved path. Evidently the magnitude of this curvature will depend on the intensity of the particular gravitational field, and, under ordinary circumstances, the deviations from rectilinear propagation are exceedingly small, so that a test by experiment is bound to fail owing to the inadequacy of the measuring instruments used in physics. But here, too, there is again one exceptional case in which stronger effects may be expected. As you are aware, the positions of the fixed stars in the sky are known to astronomers with extraordinary accuracy. Usually, however, the stars, on account of the brightness of sunlight, remain invisible during daytime; but, on the occasion of a total eclipse of the sun, very bright stars will become distinctly visible in the vicinity of the concealed disc of the sun. Now, if we observe those stars which appear immediately at the edge of the sun's disc, their rays of light, on their way to the earth, will have to pass close to the sun. They will, therefore, traverse a comparatively strong gravitational field in which they will undergo a curvature that can be accurately calculated by means of Einstein's formulæ. The result would be a displacement of the apparent position of the star in the sky as compared with the position which this particular star would occupy under normal conditions.

Led by these considerations, in the year 1914 a number of German scientists went to the Caucasus to

take the necessary photographs during a total eclipse of the sun, which was to be expected on the 24th of August. Unfortunately their efforts were frustrated by the sudden outbreak of the war, and it was not till the 29th of May 1919 that an English expedition at Sobral in Brazil had an opportunity of reaching the desired object. As a result, such a satisfactory agreement with the predictions of Einstein's theory was obtained for several stars that its confirmation can no longer be in doubt. The extraordinary importance of this confirmation is, as was stated by the German physicist, M. v. Laue, all the greater, since none of the many theories of gravitation which have been developed in connection with the "restricted" theory of relativity—quite a number of them satisfying all the older empirical facts, and some of them even correctly explaining the displacement of the spectral lines of the sun towards the red—are able to interpret this deflection of light. The fact that the general theory of relativity "starting from regions of experience so utterly different, and introducing into our physical conception of the world the boldest alterations, should have given a mathematically sufficient interpretation of this deflection, may confidently be regarded as one of the greatest triumphs of the human mind."

When, in 1915, Einstein published a coherent statement of his generalised theory of relativity, the boldness of his ideas was bound to create an enormous sensation, in particular, because, at that time, the most important experimental confirmations were still lacking. It was only natural that even well-known physicists openly declared themselves opposed to the general principle of relativity and the theory of gravitation derived from it. Einstein, however, was able to meet all objections raised against his ideas. A particularly severe attack was directed against Einstein's system in 1918 by the physicist, Philipp Lenard of Heidelberg, known by

his celebrated investigations on cathode rays. In his opinion the original special principle of relativity is well supported by experience. As long, for instance, as a railway train is in uniform rectilinear motion, there is, he admits, no possibility whatsoever of deciding whether the train or its surroundings are in motion. But then Lenard proceeds in the following words: "Now let this imaginary railway train enter on a non-uniform motion. If, in this case, by the effect of inertia everything that is in the train is smashed to bits while everything outside remains uninjured, no man in his senses will wish to draw any other deduction than this, that it was the train, and not its surroundings, which, with a jerk, changed its motion. The generalised principle of relativity, according to its simple meaning, requires that, in this case, too, we shall have to admit that, after all, it may have been the surroundings which underwent a change of velocity, and that the whole calamity in the train may thus have been nothing but the consequence of this jerk of the outside world transmitted to the interior of the train by an "effect of gravitation" caused by the outside world. To the question which suggests itself as to why the church tower close to the train had not collapsed if it had undergone the jerk together with its surroundings . . . the principle apparently has no satisfactory common-sense answer."

To these observations of Lenard, Einstein replies, in the first place, that they do not do justice to the meaning of the general principle of relativity. For this principle by no means maintains that for the interpretation of one and the same process two fundamentally different possibilities are presented to us; but it emphasises the equivalence of two ways of looking at things. "Which description one has to choose can only be decided by reasons of expediency. The fact that the tower does not collapse is, in the

second manner of representation, due to this reason that it, together with the ground and the whole earth, is falling freely in a gravitational field acting during the jerk, whilst the train is prevented from falling freely by external causes (the action of the brake). A freely falling body behaves with respect to processes in its interior like a freely suspended body removed from all external influences." Einstein then shows by a humorous counter-example "how unsuitable it is in such cases to appeal to 'common sense' as arbitrator. Lenard himself admits that so far it has not been possible to raise sound objections against the validity of the special principle of relativity. The uniformly travelling train might just as well be regarded as being 'at rest,' the track with the whole country as 'uniformly moving.' Is the common sense of the engine-driver going to admit this? He will retort that he surely has not incessantly to heat and lubricate the whole country, but the engine, and that, consequently, it must be the latter in whose motion the effect of his labours would show itself."

XI

RELATIVITY AND THE UNIVERSE

THE physical conception of the world, with the description of which we began our considerations, sought to resolve the whole of natural phenomena into a varied play of mechanical processes. Stars and atoms were to make merry in an unbounded infinite space, and eternal, irredeemable time was to render clear to us the process of their motion.

What is to remain of all this after the theory of relativity enters into the scheme of things ?

Space and time sank to shadows. Motion in itself became meaningless. The shape of bodies became a matter of view-point. And the world-æther was banished for ever. . . .

> Woe, woe !
> Thou hast destroyed
> The beautiful world
> With violent blow ;
> 'Tis shivered ! 'tis shattered !
> The fragments abroad by a demigod scattered !
> Now we sweep
> The wrecks into nothingness ;
> Fondly we weep
> The beauty that's gone !
>
> (*Translation by Anna Swanwick*)

No doubt many of us may feel like this. And yet, it is just the theory of relativity that presents us with a

view of the universe which, in its all-round beauty, leaves all earlier ones far behind.

The new system had its origin in the unexpected result of the Michelson-Morley experiment. In order to interpret it Albert Einstein created the foundation of his theory, as the first results of which the mathematical formulæ came into being for the transformation of measurements of space and time, required for the transition from one system of co-ordinates to another moving relatively to it. Now, in the study of processes of motion of any kind, measurements of space and time naturally play a most important part, because by the motion of a body we practically mean nothing else than a change of place in the course of time. Thus you will understand that, through Einstein's formulæ, the whole of mechanics had to undergo a complete modification. Before Einstein physicists had silently assumed that every measurement of magnitudes in space and time was entirely independent of the state of motion of the system of co-ordinates used. If, for instance, the motion of a falling stone on a railway truck travelling uniformly in a straight line had to be investigated, one saw nothing incongruous in an attempt to solve the problem in the following manner: First of all, one established the laws governing the motion of fall with reference to the truck, *i.e.* by means of clocks and measuring-rods one found out at what distance from the floor of the truck the stone was situated at different moments. In addition, one measured the truck's own motion with regard to the embankment by establishing, in this case, too, by means of clocks and measures of length, the change of position taken up by the truck in the course of time. In accordance with the law of inertia the stone participates unhindered in the motion of the train as long as this travels uniformly in a straight line. Consequently, we appeared to have obtained an explanation of the real motion of the stone by mathe-

matically combining into one single motion the falling motion of the stone with reference to the truck, and the motion of the train with reference to the embankment. Owing to Einstein's labours we now know that this method is utterly inadmissible. This is so because we neglected the fact that these two individual measurements of distances and times—the one on the truck and the other one on the embankment—may, by no means, be straightaway combined with each other as equivalent. The railway truck and the embankment are rather to be regarded as two systems of co-ordinates in uniform rectilinear motion relatively to each other, so that measurements of space and time in one of them have to be transformed by means of Einstein's formulæ, before they may be combined with the corresponding measurements in the other system. In this way the theory of relativity takes away the character of absolute accuracy from all statements of regularity in mechanics. They merely retain a sort of approximate validity, and can only be used with advantage when we are dealing with motions of comparatively low velocity. Einstein's formulæ show that the deviations occurring in measurements of space and time make themselves felt in a considerable degree only when we have to deal with velocities of 100,000 and more kilometres per second. This is, of course, not the case with the large majority of all mechanical phenomena, and, in particular, the motions of heavenly bodies so far known to us possess much smaller velocities. With regard to them, therefore, the special principle of relativity does not bring about any essential change. On principle, however, its application to the course of mechanical processes in nature remains necessary. Thus, for instance, a clock situated on the equator of our earth is bound to go at a slightly reduced rate as compared with a second, identically constructed clock, placed, under the same conditions, at the North or the South Pole.

There is a further reason why the fundamental laws of classical mechanics have to be regarded as approximation laws, *i.e.* laws only approximately valid. The reason is that in connection with them no attention was given to the fact that, by the presence of the heavenly bodies, the whole universe has become a single gravitational field whose intensity is subject to all sorts of variation from place to place! The space-time structure of the world depends on the nature of the gravitational field, and the former fundamental laws will have to be replaced by others which take these facts into account. You will remember the deductions to which, on account of this fact, we were led with regard to the motions of the planets—deductions most brilliantly confirmed in the case of Mercury! If, in this way, our ideas about the courses of the stars become considerably modified, the same will apply to a still greater extent to the motions of the electrons in the minute atom of the chemist. The immense velocity with which they revolve round the nucleus of the atom, charged with positive electricity, imperatively demands the application of the formulæ of the theory of relativity. It is the achievement of the physicist, Arnold Sommerfeld of Munich, to have carried out the theoretical investigations with respect to this point. He summarises his results in the words that the original absolute theory comes to grief on the facts of atomic structure, and that it will finally have to hand over to the theory of relativity the position previously occupied by itself.

The derivation of Einstein's formulæ of the special theory of relativity resulted from an endeavour exactly to interpret the phenomena of the propagation of light on the earth. Therefore it will not surprise you that in these formulæ the value of the velocity of light holds a pre-eminent position. Now, since Einstein's formulæ are decisive with regard to the course of mechanical

events, the circumstance just mentioned implies the significant fact that in general mechanics a magnitude appears which plays an important part in non-mechanical domains—in the so-called æther mechanics of older physics. Owing to this fact, the idea suggests itself that between these two great branches of physics important relations will exist which are based on the unity of all natural phenomena. The idea of such a unity had, as you know, also been the guiding thought of earlier physics. It had crystallised in a desire to find a way enabling us to recognise the essence of all physical phenomena in motions of some kind. In the course of centuries we had actually succeeded in interpreting the phenomena of heat and sound in a strictly mechanical way. The heat of a body was recognised as a result of exceedingly violent and quite disorderly motions of its molecules, and all the laws established by the theory of heat turned out to be simple deductions from the purely mechanical nature of those molecular motions. Sound is nothing but a quick succession of condensations and rarefications of the air produced by vibrating bodies, and propagated in wave-like fashion. On the other hand, the properties of light, electricity, magnetism, and radiation of heat caused serious difficulties. They led to the assumption of a world-æther, and one hoped to overcome them by an elaboration of æther mechanics. But, in spite of this conception of æther, those difficulties remained insurmountable, and they reached their climax when the hypothesis of the æther received its death-blow from the Michelson-Morley experiment. Many doubts arose as to whether one really was on the right track in this endeavour towards a mechanical explanation of all physical phenomena. Already towards the end of the nineteenth century the opinion was expressed that one would have to regard the *electrical* processes as the prototype of all physical phenomena in order to arrive at the longed-for uniform conception of the

world. And this opinion has been justified by the theory of relativity in the most brilliant manner. But, unfortunately, a thorough understanding of this most important fact can only be given by means of highly complicated mathematical explanations. However much I sympathise with your indignation at this fact—still you will have to admit, after all, that the problem of reducing the various branches of physics to a single one is, as a matter of fact, nothing but a mathematical problem. Its solution, therefore, will only be possible by mathematical methods, so that nothing will be left to you but to accept the result in its final form. The essential part of this result consists in the recognition that, by making use of the results obtained by the theory of relativity, it becomes possible to interpret the mechanical phenomena as processes of an electro-magnetic character. Not the fundamental laws of Newtonian mechanics, but the fundamental laws of modern electro-magnetic theory constitute, therefore, the foundations of theoretical physics. In the most wonderful manner the unity of our physical world has been established by this knowledge.

But now let us return to matters accessible, in all their aspects, to the non-mathematician also. From the instruction in chemistry you received in younger days you know about the law of the conservation of matter. According to it even the minutest quantity of matter can never vanish into nothingness, nor can it ever be produced out of nothing. Or, in other words, the material contents of the universe is an eternally unchanging quantity. It is only a short time ago that scientists considered the opposite view simply unthinkable. But here, too, the theory of relativity opens a powerful attack on the structure of our world of thought. The law of the conservation of matter is pitilessly scattered, and in future it will only be allowed to claim

a right to existence as a law of approximation. The reason for this is to be found in a deduction which can be derived from Einstein's formulæ. For if the quantity of energy contained in a body be increased, its mass will, according to Einstein, increase simultaneously. Take, for instance, the ivory ball on a billiard table. In order to transport it from its state of rest into that of a particular velocity, you require a very definite amount of energy. After you have found out, by experiment, the amount of this energy, you may let the ball come to rest again, and have the light of an electric arc lamp turned upon it for some time. A large part of the light will be absorbed by the ivory, and the energy contained in the ball will show an increase. If you should now measure again the energy required to obtain the velocity reached before, according to Einstein you ought to find a higher value than in a ball not subjected to radiation. For an increase of mass is said to go hand in hand with an increase of energy, and consequently the ball subjected to radiation will resist the effect of force more vigorously than the ball not subjected to radiation, and, therefore, poorer in energy. Of course we are dealing here with extraordinarily small effects which, for the time being, are not yet capable of being directly tested by experiment. Nevertheless, theoretical considerations could be advanced in confirmation of the correctness of this deduction. The inverse deduction, that the mass of a body will decrease if its energy is reduced, also proves to be tenable. Thus the strict validity of the law of the conservation of matter is evidently gone. For *every* process in nature is connected with changes of energy of some sort, and, strictly speaking, therefore, the quantity of matter in the universe must be subject to constant changes.

Immediately connected with this outcome of our reasoning is the further deduction that, at bottom, matter can be nothing but a particular form in which

energy appears to us. As a matter of fact, it would be extremely strange, if not altogether inconceivable, if changes of mass could be traced back entirely to changes in the amount of energy, but mass as such could not be reduced to energy. Thus matter and energy become inseparably united, and the law of the conservation of energy, established by Robert Mayer and Hermann von Helmholtz is raised to the position of a real universal law of physics. In the past this law of conservation offered the greatest theoretical difficulties. How extremely numerous are, after all, the forms of energy with which we have become acquainted hitherto! And what a number of energy transformations there are! The word spoken by Heraclitos of Ephesus about the eternal flow of events comes to your mind, and it appears utterly inconceivable to you how, in spite of this constant appearance and disappearance of energy, we dare to talk of the conservation of energy! The theory of relativity has supplied the solution of this puzzle. For it convinced us of the fact that all phenomena really possess an electro-magnetic character, and thus, to use the words of the physicist, Arthur Haas of Leipzig, it gave an entirely changed aspect to the principle of the conservation of energy. "According to modern views there is no transformation of energy at all. For there is only one kind of energy, namely, the energy of the electro-magnetic field. So, in reality, it is not energy which is being transformed, but, at most, the view-point of the man who observes the physical phenomena through his senses."

But however much our old conception of the world may have to be reshaped to bring it into harmony with the facts hitherto mentioned, you have not heard yet of the most surprising result of the theory of relativity. The earlier conception of the world was fond of conceiving space as boundless and infinite. Friedrich

Schiller extolled the greatness of the world in the following poem :

Upon the winged winds, among the rolling worlds I flew,
Which, by the breathing spirit, erst from ancient chaos grew ;
Seeking to land
On the farthest strand,
Where life lives no longer, to anchor alone,
And gaze on creation's last boundary stone.

Star after star around me now its shining youth uprears,
To wander through the Firmament its days of thousand years.
Sporting they roll
Round the charmed goal :
Till, as I looked on the deeps afar,
The space waned—void of a single star.

On to the realm of Nothingness—on still in dauntless flight,
Along the splendours swiftly steer my sailing wings of light ;
Heaven at the rear
Paleth, mistlike and drear,
Yet still as I wander, the worlds in their glee
Sparkle up like the bubbles that glance on a sea !

And towards me now, the lonely path I see a Pilgrim steer !
" Halt, Wanderer, halt—and answer me—What, Pilgrim, seek'st thou here ? "
" To the world's last shore
I am sailing o'er,
Where life lives no longer to anchor alone,
And gaze on creation's last boundary stone."

" Thou sailest in vain—Return ! Before thy path, Infinity ! "
" And thou in vain !—Behind me spreads Infinity to thee !
Fold thy wings drooping,
O thought, eagle-swooping !—
O Phantasie, anchor !—The voyage is o'er :
Creation, wild sailor, flows on to no shore ! "

(*Translation by Lord Lytton*)

In the garden of Epicure already the infinity of the universe has been the subject of many a learned

discussion, but only in comparatively recent times this thought became generally accepted by educated humanity. Particularly when looking at the starry sky our imagination loves to rove to limitless distances. Space, in these moments, seems infinite—infinite the number of suns. But the theory of relativity does not stop short even before this, our favourite idea. Here, too, it brings about a change ; here, too, it calls upon the ever-searching mind of man always to remain conscious of the imperfection of its achievements.

We have previously emphasised the fact that, among all kinds of surfaces, the plane occupies a privileged position. For it represents the only example of a two-dimensional continuum capable of development in a solely two-dimensional world. All other surfaces extend into the third dimension, and presuppose, therefore, the existence of a three-dimensional world. Or, in other words, all other surfaces are curved. Since your schooldays you have known that the plane is unbounded and infinite. Now imagine intelligent beings of an absolutely flat shape, *i.e.* of a strictly two-dimensional structure owing to which they will never be able to rise to the conception of a third dimension. If these beings live on a plane they will, undoubtedly, regard this world of theirs as unbounded and infinite. And their mathematicians will develop a geometry coinciding exactly with our Euclidean geometry. But now imagine the surface of a gigantic sphere. Let this sphere, too, be inhabited by strictly flat beings, infinitely small as compared with the extent of their "world," *i.e.* the surface of the sphere, so that it is absolutely impossible for any of them to undertake a "trip round the world." Obviously these beings will know nothing about the curvature of their surface world, since the conception of curvature necessarily presupposes the conception of a third dimension, which is entirely foreign to the imaginary inhabitants of the sphere. In structure and life they

are two-dimensional, and so is their mental outlook. The three-dimensional space is unknown to them, and inconceivable. So, if their philosophical and scientific observations lead them to raise the question as to the extent of their world, they will, without hesitation, declare their world to be unbounded and infinite, in the same way as if they were living on a plane surface. We, however, being three-dimensional beings, smile at their simplicity. Obviously it would be impossible to doubt the unboundedness of their world. For where could one look for a beginning, where for an end, of the perfectly self-contained surface of a sphere? But unboundedness, after all, is by no means equivalent to infinity! On the contrary! Even the surface of an unusually large sphere must by no means be called infinitely large. Thus you will see that unboundedness and infinity do not, under all circumstances, appear as indissolubly connected with each other! But will you be able to make the inhabitants of that sphere understand this fact? What would they say if some day one of them proclaimed that their world was, indeed, unbounded, but, in spite of this, by no means infinite? They would shake their heads and call him a fool, and would deride his theory as the greatest nonsense imaginable. . . .

Now, from the world of these imaginary surface-beings, return to the world of three-dimensional, unbounded space which you have always imagined as being of infinite extent. And let it be said to you that this imagination of yours was a mere illusion. That our space with all its unboundedness can never be infinitely large. And do not laugh at this theory as the fools did in the simile just quoted, who were incapable of understanding the only wise man amongst them. And try to follow, in this case too, the train of thought inaugurated by Albert Einstein.

According to Minkowski, we have to regard the world of physical phenomena into which we have been

placed as a four-dimensional continuum. Space in itself and time in itself are arbitrary, and, therefore, unjustifiable conceptions. Only space-time, the union of the two, may claim an independent existence. The relations existing between the three-dimensional space of our everyday life and the four-dimensional world of physical phenomena correspond exactly to the relations existing between the two-dimensional surface and the three-dimensional space. And just as in three-dimensional space an infinite number of the most varied surfaces is possible, thus, in the four-dimensional world, we have to reckon with the existence of an infinite number of different three-dimensional spaces. The most essential characteristic by which we are able to distinguish between surfaces is the presence or absence of curvature. Every curved surface is distinguished from the plane, the only non-curved surface, by the fact that, although a strictly two-dimensional continuum in itself, it nevertheless needs for its development the existence of a third dimension. In this sense we now have a right to advance vastly beyond our ordinary power of perception, by speaking of "curved spaces," which are to be understood as three-dimensional spaces depending for their development on the existence of a fourth dimension. But you should never try to visualise them! "Curvature of Space" is a purely intellectual conception at which we arrived by transferring the intelligible relations holding for surfaces to spaces in a four-dimensional continuum. Above all, beware of the mistake of regarding a "curved space" as a sphere! This would show that you have completely misunderstood the whole matter! For a sphere is obviously nothing but a part of space bounded by a curved surface, a statement which evidently says nothing whatever about the curvature or non-curvature of space itself! The situation is exactly the same

as if you were to describe a circle drawn on a plane as a curved surface! This illustration will, I hope, clearly show you the difference we are talking about, for a circle is, as you know, only a part of a surface limited by a curved line, and has nothing to do with the curvature of the surface itself. We can describe a circle on the surface of a sphere as well as on a plane. To have an instance of the former you only have to think of any of the circles of northern latitude on the surface of our earth. The circle on the plane circumscribes a non-curved surface, the circle of latitude on the earth a curved one.

As soon as you have recognised these facts, the question arises whether the three-dimensional space of our earlier conception of the world is a "curved space" or not. And with it goes the second question whether we have a right to claim the character of infinity for the unbounded space of our conception of the world. For this much you will easily see, that it would be impossible to speak any longer of the infinity of space if we admit "curvatures" of space of any particular kind.

But how are we to decide this new question of ours? How are we to find out if our three-dimensional world-space, embedded as it is in the four-dimensional continuum of physical phenomena, actually extends into the fourth dimension? When we were talking of two-dimensional beings, we mentioned that their mathematicians, in case their world is a plane, would have to arrive at the same Euclidean geometry with which we are familiar. But what will be the condition with regard to the two-dimensional mathematicians living on the surface of a sphere? You need not be afraid of highly learned discussions; it is possible to come to a decision on this point by very simple means. For if our beings on their spherical surface should study geometry they will designate, as the "shortest connection" between two points, a line which we, in our three-dimensional way of looking at things, would have to regard as part

of an arc. From this follows at once that, as a general rule, they will be able only to draw one single "shortest connecting line" between two points of their world, but that this will no longer be the case for such points which —as seen by us—are diametrically opposed on the surface of the sphere. For, in this case, an infinite number of "shortest connecting lines" would be possible, a fact of which you will get a good illustration if you think of the number of circles of longitude connecting the two poles of our earth. The possibility, however, of an infinite number of "shortest connecting lines" between two points contradicts the first axiom of Euclidean geometry of which we spoke on a former occasion. Hence it follows, as a matter of course, that on a spherical surface, Euclidean geometry is no longer valid, and, consequently, those flat-shaped mathematicians would have to adopt a non-Euclidean geometry.

Now, by detailed scientific investigations, it has become apparent that not only in this case, but in general, the curvature of a surface involves the non-validity of Euclidean geometry. So, if we meet anywhere with a surface on which the propositions of Euclidean geometry—either all of them or at least part of them—appear to be incorrect, we may safely draw the conclusion from this circumstance that we have to deal with a curved surface. When, a little time ago, I spoke to you of that wise man in the spherical surface-world who was trying to teach his fellow-men the theory of the finiteness of their world, I had by no means transgressed the bounds of possibility. For there is no doubt that this man would certainly be able to arrive at a correct appreciation of the finiteness of his world, if he succeeded in recognising the non-validity of Euclidean geometry in his world, and in drawing from this fact the logical conclusion as to the curvature of his world extending into the third dimension.

Obviously now the thought will occur to us whether

it should not be possible to apply to the three-dimensional continuum our latest considerations with regard to the two-dimensional continuum. Such a possibility exists, indeed, and from the results of the general theory of relativity it has already become clear that, strictly speaking, Euclidean geometry does not hold in our space. To be consistent we shall, therefore, have to regard our space as "curved," and, as Einstein was able to show, the character of this curvature corresponds to a space of finite extent. You will remember that, owing to the nature of our perceptive faculties, the geometrical structure of the world which implies, of course, that of three-dimensional space with which we are so familiar, depends on the distribution of masses in the universe, because it is determined by gravitational fields, as we know. Varying geometrical structure carries with it the validity of varying non-Euclidean geometries, and, again, the character of a particular "curvature of space" will naturally depend on the character of the non-Euclidean geometry valid in each case. The curvature of space will, therefore, not only vary everywhere on account of the varying distribution of masses, but, in addition, it will change generally in the course of time, since the suns, planets, moons, comets, and other masses of the universe are constantly moving, and produce thereby a constant change in the geometrical structure of space. You will by now have an approximate idea as to how complicated these matters are. In our representation of the subject the complications have been caused partly by the fact that we have been using —in accordance with usual ideas—the conceptions of space and time as individual and independent conceptions. Modern theoretical physics, however, works exclusively with space-time, *i.e.* the four-dimensional continuum of physical phenomena, and you will understand by now that this method is bound to carry with it immense advantages.

We have now arrived at the end of our considerations. The conception of the world as it appears in the light of the theory of relativity seems complete, at least in its essential features. In view of this new conception of the world, the wails about what has been lost or, to put it more accurately, about the great changes introduced into the conception of the world of pre-relativity days, will gradually die away. "The new system burst forth," to quote Weyl once more, "like a revolutionary storm over our old and familiar conceptions of space, time, and matter, which, up to that date, had been considered as the strongest foundations of natural science. But it did so only to make room for a clearer and deeper insight into things. . . . To-day the development, as far as the fundamental ideas are concerned, seems to have reached a certain state of finality. But no matter whether we are already standing face to face with a new definite state of things or not—in any case, we shall have to make up our minds with regard to the new ideas that have arisen. And there will be no drawing back. The development of scientific thought may once more advance beyond the limits reached now, but a return to the old and rigid system is out of the question."

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